GCDs, LCMs, and Remainders

The greatest common divisor (GCD) of two (or more) positive whole numbers is the largest number which divides evenly into both (or all) of them. For instance, the GCD of 48 and 66 is 6. We can write $\gcd(48, 66) = 6$ for short.

The least common multiple (LCM) of two (or more) numbers is the smallest positive number which is a multiple of both (or all) of them. For instance the LCM of 20 and 12 is 60. ($\text{lcm}(20, 12) = 60$.)

Examples. $\gcd(33, 27) = 3$, $\gcd(1, 20) = 1$, $\gcd(30, 42, 70) = 7$. $\text{lcm}(5, 6) = 30$, $\text{lcm}(12, 8) = 24$, $\text{lcm}(1, 4242) = 4242$, $\text{lcm}(6, 10, 15) = 30$.

1. Calculate $\gcd(2520, 7200)$.

2. The least common multiple of $12, 15, 20$, and k is 420. What is the least possible value of k?

3. How many integers between 200 and 300 inclusive leave a remainder of 5 when divided by 8?

4. What is the least possible whole number which can be multiplied by 200 such that the product is a perfect cube?

5. How many zeroes are at the end of 100! ? (That exclamation point is the factorial symbol, don't forget!)

6. (a) Find the GCD of 5! and 6!.

   (b) Find the LCM of 5! and 6!.

   (c) Find an integer n such that $n \cdot 5! = 5! + 6!$.

   (d) Find the largest prime divisor of $5! + 6!$.

7. What is the last digit in the decimal representation of

   (a) 12345^3?

   (b) 1234567^3?

   (c) 2^{314159}?

   (d) $1234567^{1234567}$?

More Eggs and Lemonade. Also, Fences.

1. You have a 4-quart jug and a 9-quart jug, and a river flowing with purest lemonade. Your recipe calls for exactly 6 quarts of lemonade. Can you measure it? How?

2. You've decided the Quarter-Hour Chicken egg you had last time was a little overcooked—you think maybe you'll try giving it 9 minutes this time. You've got a 4-minute hourglass and a 7-minute hourglass. How can you measure a span of exactly 9 minutes?

3. Working alone, Tom can whitewash a fence in 4 hours. It takes Huck 6 hours to do it. How long would it take them if they worked together?

4. Jayne writes the integers from 1 to 2000 on a piece of paper. She erases all the multiples of 3, then all the multiples of 5, and so on, erasing all the multiples of each odd prime. How many numbers are left when she finishes?

5. How many squares are there on an 8x8 chessboard? (There's more than just the 1x1 squares!)