Practice problems

19th February 2006

Differentiate the following functions:

1. $y = 5x^2 + \frac{3}{\sqrt{x}} + \sqrt{x^5}$.

2. $y = x^{2/5} + 7x^{-8/3}$.

3. $y = e^x \cdot x^5$.

4. $y = \ln x \cdot \cos x$.

5. $y = e^{5\sqrt{x}}$.

6. $y = \cos(\tan(x))$.

7. $y = \frac{\ln x}{\sqrt{x^2 - 5}}$.

8. $y = x^2 \cdot \cos(e^x)$.

9. $y = \sqrt{x^2 - 1} \cdot e^{\cos(\sqrt{x})}$.

10. $y = \frac{x^2 - 3}{5^x}$.

Find the derivative of an implicit function:

1. $x^2 + y^2 = 2xy$.

2. $y^2 x = x^3 + 3e^x$.

3. $x^{2/3} + y^{2/3} = \ln x$.

4. $e^y x = e^x$.

Find the derivative of the inverse function if $f(x)$ is given by

1. $f(x) = x^2 - 3$.

2. $f(x) = 3e^{2x}$.

Find the derivative $\frac{d}{dx} f^{-1}(a)$ for the following $f(x)$ and a:

1. $f(x) = xe^x$ and $a = 2e^2$.

2. $f(x) = x - \sqrt{x}$ and $a = 2$.

3. $f(x) = x^2 \cos x$ and $a = (2\pi)^2$.

Use logarithmic differentiation to find the derivatives of the following functions:

1. $x^{\ln x}$.

2. $(\cos x)^{\cos(x)}$.

3. $(\sin x)^{\sqrt{x}}$.

4. $(\sqrt{x})^{e^x}$.

Use linear approximations to find the approximate values of the following expressions:

1. $\sqrt{48}$.

2. $\ln(1.1e)$.

3. $\cos(\pi/2 + 0.5)$.

4. $(0.99)^{25}$.

Approximate $f(x)$ by a linear approximation:

1. $f(x) = e^{2x+1}$ at $a = -1/2$.

2. $f(x) = \ln(1 + 2x)$ at $a = 0$.

Some other problems:

1. Given that $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$, find the limit $\lim_{h \to 0} \frac{e^{ah} - c}{bh}$, where a, b, c are some numbers.

2. Given that $f'(x) = 3x - 7\ln x$, compute the derivative $\frac{d}{dx} f(\cos(x))$. (*Hint:* use the chain rule).