MATH 31A 2ND PRACTICE MIDTERM

Problem 1. For each of the following functions, compute its derivative and then use linear approximation around the given point a to estimate the value of the function.
(a) $f(x) = \sin x$, $a = \pi$, $f(\frac{5\pi}{4})$;
(b) $f(x) = \sqrt{1 + \sqrt{1 + x}}$, $a = 0$, $f(0.1)$;

Problem 2. True or False. For each of the following statements, indicate if it is true or false (you may assume that f is everywhere differentiable for all of the questions below). You will be given 4 points each for a correct answer, zero points for no answer, and penalized 4 points for an incorrect answer:

1. If $f'(x) > 0$ for all x, then $f(x)$ is increasing	
2. If $f'(a) = 0$, then f attains either a maximum or a minimum at a	
3. If f is concave up, then f' is increasing	
4. The function $\sqrt{x^2 + 1}$ is concave down	
5. The function $\sin x$ has an inflection point at π.	

Problem 3. Let $f(x) = x^3 - 3x + 7$. Find the minimum and maximum values of f in the interval $[0, 2]$.

Problem 4. Let $f(x) = \frac{1 - 2x^2}{1 - x^2}$. Sketch the graph of f, indicating all properties of the function, such as asymptotes, extreme points, minima, maxima, convexity, points of inflection and intersepts.

Problem 5. Let $y = \sin 2x - 2\sin x$. Sketch the graph of this function, inicating all properties of the function.

Problem 6. Let f be a differentiable function. Prove that if the equation $f(x) = x$ has more than one solution, then there must be a point c at which $f'(c) = 1$.

Problem 7. Find the following limits:

1. $\lim_{x \to \infty} \frac{x^4 + 9x^3 + \pi x^2 - 17x + 106}{3x^4 - 16x^3 - 149}$
2. $\lim x \to \infty \frac{5x^3 + 55x^2 + 555x + 5555}{x^8 - .003}$
3. $\lim_{x \to \infty} \sqrt{x^4 + 2x + 4} - \sqrt{x^4 - 2x - 4}$
4. $\lim x \to \infty \frac{\sin^2 x}{x^2}$