Practice Midterm 2

Math 61, Section 2, Fall 2009

- The Midterm will take place **12:00-12:50 pm on Monday, May 18** in **Boelter 3400**. Note this location is *not* our regular class meeting room.
- You will need to bring a pencil and your student ID card to the midterm. No other materials or devices will be allowed.
- The following sections from *Discrete Source* will be emphasized on the midterm: Generalized Permutations and Combinations, Binomial Coefficients and Combinatorial Identities, Pigeonhole Principle, Recurrence Relations, Solving Recurrence Relations, Graph Theory: Introduction, Paths and Cycles, Hamiltonian Cycles and the Traveling Salesperson Problem, A Shortest-Path Algorithm, Representations of Graphs, Isomorphisms of Graphs, Planar Graphs.
- Disclaimer: questions on the practice midterm *may not* be similar to those on the actual midterm!
- As usual, you may leave numerical answers in *un-simplified* form – that is, a form which you could plug into a calculator using only numbers and "+", "−", "∗", "/", "!", and "ˆ," and arrive at the numerical solution.

1. Dijkstra's algorithm can fail if some of the edge weights are negative. Give an example showing Dijkstra's algorithm failing for a graph with some negative edge weights.

2.

(**a**) Find the general solution to the recurrence relation: $a_n = 3a_{n-1} - 2a_{n-2} - 2n$

(**b**) Find the solution of the recurrence relation in (a) subject to the initial conditions: $a_0 = 1$, $a_1 = 2$.

(**c**) Find the general solution to the recurrence relation: $a_n = 3na_{n-1}$

(**d**) Find the solution of the recurrence relation in (c) subject to the initial condition: $a_0 = 2$.

3. Find the coefficient of x^6y^{29} in the expansion of $(3x - y)^{35}$.

4.

(**a**) The four kids Alf, Borat, Chauncey, and Dora have 36 chocolate bars, which we consider identical, in a pile. How many ways can the 36 bars be distributed among the four kids if Alf gets at least 5 bars?

(**b**) How many ways can the 36 bars be distributed among the four kids if Alf gets at least 5 bars *but no more than* 15 bars?

(**c**) Now Alf, Borat, and Dora decide to head to the cloning laboratory so that we end up with 8 identical Alfs, 20 identical Borats, and 16 identical Doras, along with Chauncey. How many different ways can we line them all up in a line?

5. Let G be the graph with vertices $V = \{a, b, c, d, e\}$ and with adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

where the vertices are ordered alphabetically in the matrix.

(**a**) Is G bipartite? Explain your answer.

(**b**) Is G planar? Explain your answer.

(**c**) Does G have a Hamiltonian cycle? Explain your answer.

(**d**) What is the least number of edges we can add to G so that it will have an Euler cycle? Explain your answer.

(**e**) Give a function $f : V \to V$ that is part of an isomorphism f, g from G to itself, and is not exactly the same as the identity function on V.

6. Assume that G_1 is a connected graph that is isomorphic to the graph G_2. Prove that G_2 is also connected. (That is, prove that connectivity is an invariant.)