Practice Midterm 1

Math 61, Section 2, Fall 2009

- The Midterm will take place **12:00-12:50 pm on Monday, April 20** in **Boelter 3400**. Note this location is *not* our regular class meeting room.
- You will need to bring a pencil and your student ID card to the midterm. No other materials or devices will be allowed.
- All of the following sections from *Discrete Source* are fair game for the midterm: Mathematical Induction, Sets, Functions, Sequences and Strings, Relations, Equivalence Relations, Matrices of Relations, Counting Methods: Basic Principles, and Permutations and Combinations.
- Disclaimer: questions on the practice midterm *may not* be similar to those on the actual midterm!

1. Consider 5 distinct math books and 4 distinct computer science books, to be placed on a shelf. You may leave answers in un-simplified form.

(**a**) How many different ways can we place the books on the shelf?
(**b**) How many different ways can the books be placed on the shelf if we start with a math book, then place a computer science book to its right, and continue alternating types of books to the right until we run out?
(**c**) Suppose we place the books, then an earthquake occurs and 3 books fall onto the floor. How many different possible combinations of books can end up on the floor?

2. Prove the following:
(**a**) Assume Q is a transitive relation. Whenever $(x, y) \in Q \circ Q$, then $(x, y) \in Q$.
(**b**) Assume T is a relation and that $T \circ T$ is a subset of T. Then T is transitive.

3. Let $X = \{a, b, c, d\}$ and let R be a relation on X given by $R = \{(c, a), (a, d), (c, d), (b, b)\}$.
(**a**) Is R a partial ordering on X? Justify your answer.
(**b**) Give the matrix representation of R.
(**c**) Use the matrix of R to calculate the matrix of the relation $R \circ R$.

4. Prove, using induction, that the following is true for all $n = 1, 2, 3, \ldots$:

$$\frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \cdots + \frac{1}{(n+1)^2 - 1} = \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$$

5. Let S be the set of all binary strings of length 6. (for example, 110111 is in S.)

(**a**) What is the value of $|S|$?
(**b**) What is the value of $|\mathcal{P}(\mathcal{S})|$? You may leave your answer in un-simplified form.
(**c**) Let E be the relation on S defined by: sEt if, reading left to right, s contains its first "1" in the same position as t. Is E an equivalence relation on S? Justify your answer.