

Show all work clearly and in order. No calculators, cell phones, or any other electronic devices, and no books or notes are allowed. You have 20 minutes to take this 20 point quiz.

1. (10 points) You have 80 pebbles, 42 of which are white. Show that if you line all 80 pebbles up left to right in any order, you must end up with at least two white pebbles which are three pebbles apart (i.e., exactly two pebbles are between them).

PLACE THE PEBBLES AND NUMBER THEM BY PLACEMENT, 1-80.

LET THE WHITE PEBBLE NUMBERS BE  $w_1, w_2, \dots, w_{42}$ .

NOW CONSIDER THE ELEMENTS:

$w_1, w_2, \dots, w_{42} \leftarrow 42$  DISTINCT ELEMENTS IN  $\{1, \dots, 80\}$

$(w_1+3), (w_2+3), \dots, (w_{42}+3) \leftarrow 42$  DISTINCT ELEMENTS IN  $\{4, \dots, 83\}$

WE HAVE 84 ELEMENTS IN  $\{1, \dots, 83\}$ . THUS BY THE PIGEONHOLE PRINCIPLE, THERE MUST EXIST  $i, j$  S.T.  $w_i = w_j + 3$ , SO WHITE PEBBLES  $i$  AND  $j$  ARE 3 APART. (NOTE: SINCE  $w_i \leq 80$  AND  $w_j = w_i - 3$ , IT FOLLOWS THAT WE MUST HAVE  $w_j \leq 77$ .)

2. (10 points) Find the general solution to the recurrence relation:  $a_n = 3a_{n-1} + 18a_{n-2} + 10$

1) SOLVE  $\textcircled{H}$ :  $a_n = 3a_{n-1} + 18a_{n-2}$   $\textcircled{I} \uparrow$

$$\Rightarrow t^2 - 3t - 18 = 0$$

$$(t+3)(t-6) = 0$$

$$\Rightarrow t = -3, 6 \Rightarrow U_n = b(-3)^n + d(6)^n$$

2)  $g(n) = A$  (SAME FORM AS INHOMOGENEOUS TERM + LOWER POWERS OF  $n$ )

3)  $g(n) = A$  (UNCHANGED BECAUSE OF NO CONFLICT W/  $U_n$  TERMS)

4) PLUG  $g(n)$  INTO  $\textcircled{I}$  AND SOLVE FOR  $A$ :

$$\Rightarrow A = 3A + 18A + 10$$

$$-20A = 10$$

$$A = -\frac{1}{2}$$

$$5) W_n = U_n + g(n) = \boxed{b(-3)^n + d(6)^n - \frac{1}{2}}$$