Math 61, Section 2, Spring 2009
Week 2 Quiz, 04/09/09

Name: KEY

Show all work clearly and in order. No calculators, cell phones, or any other electronic devices, and no books or notes are allowed. You have 15 minutes to take this 20 point quiz.

1. (*10 points*) Prove, using induction, that the following is true for every positive integer $n \geq 1$:

$$1 + 3 + 5 + \cdots + (2n-1) = n^2 \tag{1}$$

BASIS: $n=1$: $1 = (1)^2$ ✓

INDUCTIVE: GIVEN: $1+3+\cdots+(2n-1) = n^2$

SHOW: $1+3+\cdots+(2n-1)+(2n+1) = (n+1)^2$

$(n+1)^2 = n^2+2n+1$

$= 1+3+\cdots+(2n-1)+2n+1$ BY GIVEN

✓

2. (*10 points*) Prove, using induction, that the following is true for every positive integer $n \geq 1$:

$$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \geq \frac{1}{2n} \tag{2}$$

BASIS: $n=1$: $\frac{1}{2} \geq \frac{1}{2(1)}$ ✓

INDUCTIVE: GIVEN: $\frac{1\cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots (2n)} \geq \frac{1}{2n}$

SHOW: $\frac{1\cdot 3 \cdots (2n-1)(2n+1)}{2 \cdot 4 \cdots (2n)(2n+2)} \geq \frac{1}{2n+2}$

$\frac{1\cdot 3 \cdots (2n-1)(2n+1)}{2 \cdot 4 \cdots (2n)(2n+2)} \geq \frac{1}{2n} \cdot \frac{(2n+1)}{(2n+2)}$ BY GIVEN

$\geq \frac{1}{2n+2}$, SINCE $\frac{2n+1}{2n} = 1 + \frac{1}{2n} > 1 \quad \forall n \geq 1$

✓