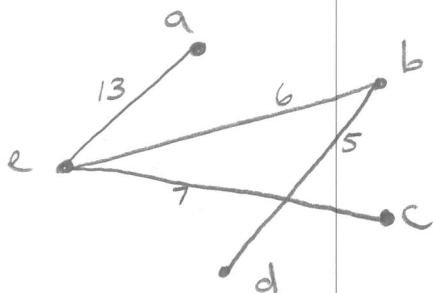
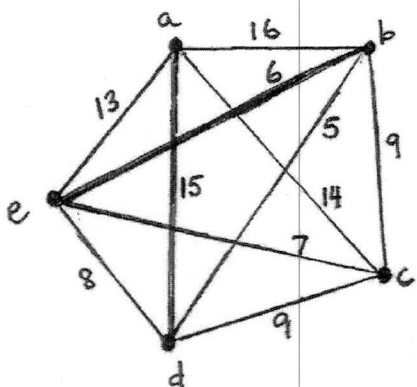


Show all work clearly and in order. No calculators, cell phones, or any other electronic devices, and no books or notes are allowed. You have 20 minutes to take this 20 point quiz.

1. (10 points) Consider the weighted graph given below. Find a minimal spanning tree.



Use PRIM'S ALGORITHM.

START AT VERTEX a.

AVAIL. EDGES	WEIGHT
(a, b)	16
(a, c)	14
(a, d)	15
(a, e)	13

ADD EDGE (a, e) AND VTX. e.

AVAIL. EDGES	WEIGHT
(a, b)	16
(a, c)	14
(a, d)	15
(e, b)	6
(e, c)	7
(e, d)	8

ADD EDGE (e, b), VTX. b.

AVAIL. EDGES	WEIGHT
(a, c)	14
(a, d)	15
(e, c)	7
(e, d)	8
(b, c)	9
(b, d)	15

ADD EDGE (b, d), VTX. d.

AVAIL. EDGES	WEIGHT
(a, c)	14
(b, c)	9
(d, c)	9
(e, c)	7

ADD EDGE (e, c), VTX. c AND STOP.

2. (10 points) Prove the theorem given in class: If T is a full binary tree with i internal vertices, then it has $2i + 1$ total vertices.

EVERY INTERNAL VERTEX IN A FULL BINARY TREE HAS 2 CHILDREN, SO THERE ARE $2i$ CHILDREN. EVERY VERTEX IN A TREE IS A CHILD EXCEPT THE ROOT. SO WE HAVE $2i$ CHILDREN + 1 ROOT = $2i + 1$.