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Ribbon tile invariants. (English)

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In this fundamental paper, the problem of whether or not a polyomino (i.e., a simply connected region in the plane that is composed out of unit squares centered at integer points) can be tiled using tiles from a given set is addressed. Nowadays, there are basically two general techniques to attack this problem: colouring arguments and Conway group analysis. The author enlarges this set of tools. Given a finite set $\mathbf{T}$ of tiles and a set $\mathcal{B}$ of regions that are tileable by $\mathbf{T}$, he introduces the tile counting group $\mathbb{G}(\mathbf{T}, \mathcal{B})$ as a naturally defined quotient group in the group of all formal integer linear combinations of $\mathbf{T}$. His main theorem is the computation of the tile counting group when $\mathbf{T}$ is a set of, what the author calls, ribbon tiles. A ribbon tile can be considered as a (connected) skew Young diagram which does not contain any 2×2 square. (Other terms used in the literature are "ribbons" or "rim hooks.") In this case, the tile counting group is given explicitly by a certain set of generators. This theorem enables the author to establish remarkable characterizations for a large family of regions which tell whether or not these regions can be tiled by using certain ribbon tiles. Interestingly, one of the main tools in the proof of his theorem is the well-known rim hook bijection which maps rim hook tableaux to certain skew standard Young tableaux.

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Keywords : polyomino tilings; tile invariants; Conway tiling group; rim hooks; ribbon hooks; Young diagrams; Young tableaux; rim hook bijection; symmetric group
Classification :
∗05B45 Tessellation and tiling problems
20C30 Representations of finite symmetric groups
52C20 Tilings in 2 dimensions (discrete geometry)
05E10 Tableaux, etc.

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