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Partition bijections, a survey. (English summary)

Ramanujan J. **12** (2006), *no. 1*, 5–75.

This paper undertakes a monumental task: to present a reasonably coherent account of partition bijections. The difficulties for the author mostly arise from the fact that the literature is strewn with single bijections given here and there stretching over a century, and often each bit seems only distantly related to any other bit.

Thus we owe a large debt to the author, who has in this 71-page paper provided us with a readable account of the many techniques that have appeared, and has given us a good understanding of the interrelationships involved. The extensive bibliography of 130 entries gives the reader plenty of direction in using the literature.

The paper begins with a helpful introduction containing some history, with a guide to the author's selection of material, structure, language, and notation. The next eight sections have the titles (2) Basic results, (3) Euler's theorem, (4) Partition theorems of Lebesgue, Göllnitz, and Schur, (5) Euler's pentagonal number theorem, (6) Jacobi's triple product identity, (7) Rogers-Ramanujan identities, (8) Involution principle and partition identities, (9) Miscellanea. The paper concludes with final remarks, which provide further commentary on each of sections 2–8.

This is a truly important contribution. The author's presentation is lucid, often clarifying or improving the original work and providing new insights. In addition a number of "exercises" are included; the solutions to some of these may well be independently publishable.

We might finish by quoting from the paper under review:

"Let us conclude this brief excursion into the past, present and future of 'constructive partition theory' on an optimistic note. We believe there is a great deal to be done before we reach a better understanding of the combinatorial nature of partition identities. We hope this survey will provide guidance and the foundation of future investigations. As D. J. Kleitman once said, 'Combinatorics will survive as long as mathematics does'. To paraphrase this, partition theory will survive as long as combinatorics does, and we believe its future is as bright as it was imagined by Sylvester so many years ago. . . ."

Reviewed by *George E. Andrews*

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