# ERRATUM FOR LONG CYCLES IN ABC-PERMUTATIONS

IGOR PAK, AMANDA REDLICH

This is a correction to Lemma 2, which leads to a cleaner proof of the main theorem.

Let $g(n,k) = 1/k^2$. Let $\epsilon(n,k) = f(n,k) - g(n,k)$. Then

$$1 - \sum_{k=2}^{n} \mu(n,k) f(n,k) = 1 - \sum_{k=2}^{n} \mu(n,k) g(n,k) - \sum_{k=2}^{n} \mu(n,k)\epsilon(n,k).$$

Since $g(n,k)$ is multiplicative, we know $1 - \sum_{k=2}^{n} \mu(n,k) g(n,k) = \prod_{p \text{ prime}} (1 - 1/p^2)$. So all that remains is to bound $\sum_{k=2}^{n} \mu(n,k)\epsilon(n,k)$.

Note that $\epsilon(n,k)$ is always positive, and is either

$$\begin{aligned}\frac{(n/k+1)(n/k+2)}{n(n+1)} - 1/k^2 &= \frac{n^2+3nk+2}{n(n+1)k^2} - 1/k^2 = 1/k^2 + \frac{3k-1}{k^2(n+1)} + \frac{2}{n(n+1)k^2} - 1/k^2 \\ &= \frac{3}{k(n+1)} - \frac{1}{k(n+1)} - \frac{1}{k^2(n+1)} + \frac{2}{n(n+1)k^2} \le \frac{5}{k(n+1)}\end{aligned}$$

or else

$$\begin{aligned}\frac{(\lfloor \frac{n}{k} \rfloor)(\lfloor \frac{n}{k} \rfloor + 1)}{n(n+1)} - \frac{1}{k^2} &\le \frac{(\frac{n}{k})(\frac{n}{k}+1)}{n(n+1)} - \frac{1}{k^2} = \frac{n^2+kn}{k^2 n(n+1)} - \frac{1}{k^2} \\ &= \frac{k-1}{(n+1)k^2} = \frac{1}{(n+1)(k)} - \frac{1}{(n+1)k^2} \le \frac{5}{k(n+1)}.\end{aligned}$$

Therefore we can bound

$$\begin{aligned}\sum_{k=2}^{n} |\mu(n,k)\epsilon(n,k)| &\le \sum_{k=2}^{n} \frac{5}{k(n+1)} \\ &= \frac{5}{n+1} \sum_{k=2}^{n} 1/k \to \frac{5 \ln n - 1}{n+1} \to 0\end{aligned}$$

As $n$ goes to infinity,

$$1 - \sum_{k=2}^{n} \mu(n,k) f(n,k) \to 1 - \sum_{k=2}^{n} \mu(n,k) g(n,k).$$

Since

$$1 - \sum_{k=2}^{n} \mu(n,k) g(n,k) = \prod_{p \text{ a prime}} g(n,k) = 6/\pi^2,$$

this completes the proof.

*Date*: February 15, 2010.