Opinion

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Will Mathematics Survive?[1] Report on the Zurich Congress

V. I. Arnold

Every four years mathematicians from around the world gather together at their International Congress of Mathematicians to find out who are the new champions (as in the Olympic Games or the "Hamburg accounting" described by Shklovskii).

In August 1994 the Congress took place in Zurich. (This is the third time it has been held there; the very first Congress was in Zurich, in 1897.) The Congresses are not numbered, because not everyone agrees on which past Congresses should be included.

In the years prior to the Congress, a specially selected International Program Committee (whose membership is secret until the opening of the Congress) chooses invited speakers. This year's invited talks were 16 plenary talks of one hour and 156 sectional talks of 45 minutes. There are 19 sections (mathematical logic, algebra, number theory, ... to history of mathematics and mathematical pedagogy), and the talks were given in seven halls simultaneously. Every day one could attend six talks. An invitation to speak at the Congress is considered a great honor; it can be (unfortunately?) very important for the career of a mathematician in a very strained world job market.

This year among the 16 plenary speakers 3 were from the Russian school of mathematics (in Kyoto, 1990, there had been 4 out of 15). Among the 156 sectional speakers I counted 14 from the Russian school (in Kyoto, 19 out of 139). In these counts I did not consider the present place of employment. Why our standing has dropped from 14% to 9% in four years — say, by a third — remains to be explained.

At the Congress the names of those to be honored with Fields Medals were announced: J. Bourgain (France and and USA), E. Zelmanov (Russia and USA), J.-C. Yoccoz (France), P.-L. Lions (France). These medals, awarded to mathematicians aged at most 40, are often compared to the Nobel Prizes (there is no Nobel Prize for mathemat-

V. I. Arnold

If we follow Vladimir Nabokov, who said the right way to introduce Tolstoy is just to say, *"Anna Karenina,"* we may identify *V. I. Arnold* by mentioning ABC, ADE, KAM, Liouville tori, the 13th and 16th Hilbert problems. . . . Then there is "strange duality" of Lobachevskii triangles, which may be the first appearance (1974) of what is now called mirror symmetry. For recent news of him, in the form of plane curves and wave fronts, see the 21st volume of *Advances in Soviet Mathematics* (American Math. Society, 1994) and the 5th volume of the University Lecture Series (ibid.).

[1] This is a translation of an article in the general-circulation magazine *Eureka*, somewhat amplified by the author.

ics). The comparison is undeserved: unlike the Nobel Prizes, the Fields Medals pass by many of the truly outstanding people, and in particular Russians.

To give three medals at once to representatives of the French mathematical school, and all three of them noted for the art of manipulation of inequalities, is hardly a help to the international prestige of French mathematics[2] — particularly coming at a time when the president of the International Mathematical Union (the organization naming the Fields Committee) was a well-known French analyst. It is hard to affirm a causal relation between these circumstances; that would imply a quite unlikely degree of corruption. But it is one more reminder how invalid is the comparison of Fields Medals to Nobel Prizes. The Nobel Committee asks the opinion of a much wider group of specialists than the Fields Committee does, and as a rule it does not leave itself open to suspicions (even unfounded) of the sort pointed out above. (I am afraid I heard such suspicions expressed by all too many participants at the Congress, from many countries and specialties.)

This year, many at the Congress reacted to the names of the medalists with, "Who is he?" Well, according to Plutarch, for a young person a medal is not a reward but a deposit toward future achievement. Let us hope that this year's laureates go on to achievements justifying the investment.

I counted 10 women among the invited speakers. This included — a special honor — plenary talks in the opening session (M. Ratner, "Interaction between ergodic theory, Lie groups, and number theory") and the closing session (I. Daubechies, "Wavelets and other methods of localization in phase space"). Outside of the regular program was a special "Emmy Noether Lecture" by a woman mathematician. This was delivered by Academician O. A. Ladyzhenskaya (St. Petersburg).

At the General Assembly of the International Mathematical Union (a sort of mathematicians' UN) held in Lucerne just before the Congress, the American delegation proposed to "increase the representation of women and to achieve a better balance of ethnic groups" among invited speakers. This proposal was rejected by the Assembly as a hidden insult to both women and ethnic groups. One Assembly delegate remarked, "It is strange that, contrary to their usual practice, Americans didn't mention sexual minorities."[3]

[2] Here I take a view contrary to that of P. Cartier.

[3] The remark was found offensive by many. Let me try to explain why. It was heard as a witty *reductio ad absurdum*: the speaker meant to thwart efforts for participation of women and minorities by putting them in analogy with efforts for participation of homosexuals. The wit relies, then, on the listener finding absurd the notion of defending homosexuals' participation. The insult to homosexuals was incidental to the speaker's intent to laugh down the American proposal, but it hurt. — Editor's Note.

Registered Participants in the Zurich Congress: 2370 (Down from 4102 at Kyoto)

Breakdown: from	USA	443
	Switzerland	229
	Japan	228
	Russia	194
	Germany	191
	France	162
	Canada	82
	...	

The Assembly did adopt a resolution to make public when appointed the identity of the Chair of the Program Committee (so that this person could come under pressure). Delegates from developing countries hoped that the entire deliberations of the Program Committee and the sectional committees would be made public. Countries without an established mathematical tradition tend to be represented in the Assembly by politicians rather than mathematicians. The resolution by the Assembly carries a very real danger which could have grave consequences for the world mathematical community.

The difference between the tenth best, who will be invited, and the eleventh, who will not, is very small.

Each sectional subcommittee ("panel") is supposed to name ten or so as best among the twenty or so most active people in its area (who did not give invited talks at previous congresses). The difference between the tenth best, who will be invited, and the eleventh, who will not, is very small. An attempt to make discussion of this difference public will only add to the weight of extra-scientific considerations (representation of different countries, sexes, nationalities, etc.). The relatively few women speakers at the Zurich Congress won this honor in fair competition with men, without allowances being made.[4]

The Assembly gave special attention to the public image of mathematics.

At the beginning of this century a self-destructive democratic principle was advanced in mathematics (especially by Hilbert), according to which all axiom systems have equal right to be analyzed, and the value of a mathematical achievement is determined, not by its significance and usefulness as in other sciences, but by

[4] Certainly. But this fair competition occurred only after vocal demands that Congresses stop underrepresenting women. See Lenore Blum, *Mathematical Intelligencer* vol. 9, no. 2 (1987), 28–32. — Editor's Note.

its difficulty alone, as in mountaineering. This principle quickly led mathematicians to break from physics and to separate from all other sciences. In the eyes of all normal people, they were transformed into a sinister priestly caste of a dying religion, like Druids, parasitic on science and technology, recruiting acolytes in the mathematical schools by Zombie-like mental subjection.

...the angel of topology and the devil of abstract algebra fight for the soul...

Bizarre questions like Fermat's problem or problems on sums of prime numbers were elevated to supposedly central problems of mathematics. ("Why *add* prime numbers?" marvelled the great physicist Lev Landau. "Prime numbers are made to be multiplied, not added!")

Unfortunately, mathematicians themselves contributed a lot to entrenching this image of their science, especially to entrenching the myth of its inaccessibility to the uninitiated.

Hermann Weyl, one of the greatest mathematicians of our times (who worked, by the way, in Zurich), said, "In these days the angel of topology and the devil of abstract algebra fight for the soul of each individual mathematical domain."[5]

In the first half of the century, the devil was winning. A special "axiomatic-bourbakist" method of exposition of mathematics was invented, to make it perfectly pure. For example, suppose we are saying that the value of a product is unaffected by the order of the factors. If we want, we can define multiplication using "rules for addition of columns." That the answer is independent of the order of multiplication can be deduced purely formally from one of these rules without knowing anything of the content of the operation of multiplication. This formal proof is urged on students by the criminal bourbakizers and algebraizers of mathematics.

If we didn't know the content of the idea of addition — if we had not first counted apples or pebbles or (with Mayakovskii) cigarette butts or locomotives — clearly we could not understand the formal proof. It is convincing only to those who have undergone a distinctive algebraic perversion of the mind, and is useless for teaching and for all applications. The grave consequences of this perversion for mathematical education in Russia and elsewhere are well known. Whole generations of mathematicians came along knowing no other style of mathematics — and of course, no other sciences. Avenging their humiliating experience in school, leaders of most countries, like the proverbial pig under the oak, planned and implemented the extermination of mathematics. According to American data, this process will take 10 to 15 years.

[5] "Invariants," *Duke Math. J.* 5 (1939), see p. 500.

Their logic is simple. England got nothing for Newton inventing the calculus; or Germany for Leibniz devising the notation the whole world uses; or France for Poincaré creating modern mathematics (topology and dynamical systems), which is indispensable in, say, radio. The American leaders, in accordance with the opinion of their voters and taxpayers, are not about to fund fundamental research (such as mathematics) unless it is proved that countries where fundamental research gets funding (Russia, France) are better off than those where it gets almost none.

Selfish calculations by the separate states lead them to scrap fundamental research needed by humanity as a whole (especially mathematics) as soon as military confrontation ends: no star wars, no supercollider, no mathematics.

The return of contemporary mathematics to the mainstream of natural science, seen everywhere over the last few decades, has not yet been reflected in the conception of mathematics and mathematicians held by the "person in the street." This is true for both "pure" and "applied" mathematics.

There is no scientific distinction between pure and applied mathematics, just a social one.

For that matter, there isn't a scientific distinction between pure and applied mathematics so much as a social one. The "pure" mathematician is paid to do mathematics, the "applied" mathematician to solve a particular problem. If a number-theorist were paid to solve the Fermat problem, then number theory would be an applied field — like the theory of Galois fields and curves over finite fields, where research is funded by the CIA, KGB, and similar agencies for purposes of cryptography.

Columbus as he set sail was in the position of the applied mathematician: he was being paid to solve a specific problem, finding a passage to India. The discovery of the New World, however, was more analogous to pure mathematics. Coastwise navigation brought the Spanish economy much more short-term benefit than the unprofitable voyages of Columbus.

Contemporary applications of mathematics, including "computer science" and applications of computers, draw on reserves of wealth accumulated by "pure" mathematics of previous generations. In contrast to geography, discoveries on the order of Columbus's are still possible — and happen yearly.

Explaining these discoveries to the uninitiated is, to be sure, not easy. Behold the Princeton mathematician John Conway come to address this problem before an audience of three thousand in the Kongressenhaus of Zurich. He appears on the brightly lit dais in shorts, sandals, and windbreaker. "Nobody knows," he says, "how to fill our ordinary three-dimensional space as densely as possible with identical spheres. It is supposed that the best way is to pack the balls in rows and layers, in the way I'll show you now." The lecturer pulls out of his windbreaker pocket something all crumpled like a handkerchief. This turns out to be a piece of some kind of plastic that quickly uncrumples and becomes a blue ball the size of a baby's head. "Let us put next to it a few more balls," says Conway and takes about ten more out of the same pocket. He lays them adjacent on the table so they form a lattice of equilateral triangles. "Now," says the lecturer, "let us put another layer on top" — and fishes in another pocket of the windbreaker for red balls. When the third layer (of green balls, from a third pocket of the windbreaker) has been put in place, everyone clearly understands the layered packing of all of space.

"Now I don't need this ball any more," says Conway, and takes a ball from the top of his pyramid and hurls it into the hall somewhere between the twentieth and the fortieth row. "I don't need these either," and he goes on throwing colored balls to all corners of the hall. When all the balls have been thrown (and caught with a happy shout by somebody in the audience), Conway remarks, "Now I don't need the windbreaker either," and takes it off and throws it on the floor. The shorts stay on throughout the lecture.

Speakers are trying to show what great scientists they are more than to impart something to the audience.

Eccentric as it was, Conway's was one of the most understandable talks in the Congress.

The trouble is the progressive conversion of congresses into Reputation Fairs: speakers are trying to show what great scientists they are more than to impart something to the audience, and they think their purpose is served by incomprehensible lectures. (This is especially so in the section talks.)

In my opinion, the best talks at the Congress were the one by Clifford Taubes (Harvard, USA) surveying the geometry of 4-dimensional manifolds (in connection with physics of gauge fields) and the one by Jürg Fröhlich of Zurich about his recent theory of the Quantum Hall Effect.

The topology of three or four dimensions has proved to be more complicated than either the topology of curves and surfaces or that of five or more dimensions. For example, only in 4 dimensions are there "fake Euclidean spaces," topologically equivalent to ordinary space but not admitting a global smooth coordinate system.

All these fake 4-spaces have, by the way, a nice description in terms of dynamical systems: they are the orbit spaces of certain smooth vector-fields (with no zeros) in the usual Euclidean 5-space. Yet, as far as I know, no one has ever written such a vector-field explicitly. May its components be elementary functions? polynomials?

Three interesting talks were devoted to the theory of *mirror symmetry*, a striking connection, recently discovered by physicists, between apparently unrelated mathematical theories, belonging to algebraic geometry, singularity theory, topology, and combinatorics of convex polyhedra. Many assertions of this theory remain so far only conjectures (corroborated by vast experimental data and by the equality of integers with a great many digits occurring in the different theories). Yet in the talks of M. Kontsevich ("Homological algebra of mirror symmetry"), of A. B. Givental' ("Homological geometry and mirror symmetry"), and of D. R. Morrison ("Mirror symmetry and moduli spaces of conformal field theories"), one could get a relatively harmonious view of a theory to come.

In the talk by Föllmer (Bonn) on financial mathematics, it was pleasant to hear about "Russian options," introduced by Shiryaev and Shepp. I had hitherto heard only of European and American options.

I was greatly impressed by the short description of the work of Nevanlinna Prize winner A. Wigderson, given at the opening session by Yu. Matijasevich (St. Petersburg). This is about new ideas in complexity of solution of mathematical problems and applications of probabilistic ideas to proof theory. Finally, we have the possibility in rigorous mathematics of finding proofs that are correct, not with certainty, but with an extremely small probability of error (say, 10^{-500}).

Everyone knows that some problems have solutions that are easy to verify but hard to find. An example is the decomposition of an integer into prime factors. If a factor is known, then one can verify the divisibility quite fast (even if the dividend has two hundred digits and the divisor one hundred). Yet finding a factor is very difficult: one has to just go through the possibilities, and the time required is enormous (growing exponentially with the number of digits of the given number). Problems of exponential complexity are out of reach of computers in practice, and will remain so however the technology may be perfected. This makes factorizations usable in schemes for transmitting secret information over public communication channels.

However, it remains an unproved conjecture that these classes really are more than polynomially complex (though a list has been assembled of thousands of problems, each of which is known to be "hard" if any is). The new advance is the elaboration of a broad theory based on the still-unproved hypothesis of the existence of hard problems. Especially interesting is the study of randomized algorithms — algorithms including random tests — and of possible means of derandomizing them — that is, replacing their stochastic elements by pseudorandom number generators. It is shown in particular that an arbitrarily complex proof can be made verifiable arbitrarily fast.

All this activity based on an unproved conjecture (with sometimes paradoxical consequences) reminds me of the work of Lobachevskii, who constructed a beautiful theory of his geometry, undeterred by having an unproved hypothesis at the foundation. Now we know that there are two geometries, one where Lobachevskii's hypothesis is satisfied and one where it is not. They simply describe the geometry of different surfaces.

American universities boast about what famous Russian mathematicians they have rejected.

It seems doubtful that there can be a mathematics that contains exponentially hard problems (impossible to solve without combinatorial search) and another that does not. In any case, various aspects of deterministic, randomized, and derandomized algorithms provided many interesting lectures at Zurich (the section on computer science).

Most talks at the Congress, however, were like sermons. The lecturers plainly didn't expect that listeners would understand anything. Sometimes they went so far as to state obviously false theorems to the respectfully silent auditorium. The sermon mood was so pervasive that most of the introducers didn't even ask for questions at the end. And when some old-fashioned professors, like J. Moser (Director of the Mathematical Institute of ETH Zurich, the principal mathematical center in Switzerland), did urge people to ask questions, very few listeners overcame fear of exposing their ignorance sufficiently to do so.

The talks differed from sermons, however, in not being free. For those not registered as participants, the fee to attend a talk was considerable, as for a concert or a play.

I take pleasure in reporting that representatives of the Russian school generally were on the more comprehensible side. It is part of our tradition that a survey talk should emphasize new ideas and illuminating examples and not technical details.

I find rather worrisome the distinct shift in interests of our younger researchers (especially those working in the West) from directions long pursued by us to those fashionable in the USA. Such a shift of interests (doubtless related to the difficult conditions of job-hunting in American universities, some of which boast about what famous Russian mathematicians they have rejected) is inevitably negative. World leaders in one field leave it to race in a pack of jostling competitors following some other leader. Could this explain the distinct decrease in the proportion of our mathematicians among speakers at Congresses?

It is a pleasure to note also the large number of young Congress participants, including graduate students, from Russia and other countries of the former Soviet Union. Their attendance was made possible by generous support from the Swiss Organizing Committee of the Congress and the Soros Foundation.

Swiss mathematicians did everything possible to make our stay pleasant: participants were offered trips all over Switzerland (Lucerne, Interlaken, Bern, etc.), trips to the mountains (to Rigi Kulm overlooking the Vierwaldstätter See), to the Rhine waterfall (comparable to Niagara), concerts of classical and folk music. I was impressed by the small and little-known Bürlet art gallery in Zurich — Rembrandt and Franz Hals, El Greco and Goya, Canaletto and Tiepolo, Greuze and Ingres, Corot and Courbet, Cézanne, van Gogh, Matisse, Pissarro, Picasso.

After the tiring Congress, I spent a day at the home of my old friend A. Haefliger near Geneva. We climbed from 1500m to 3000m in the mountains near the Rhone valley, about halfway between the Jungfrau and the Matterhorn, and I got to swim in a glacial lake. On the return I picked mushrooms, sorrel, blueberries, and wild strawberries, and made my hosts a dinner from these gifts of nature (overcoming their doubts as to their edibility). The next day I returned to Moscow.

Steklov Mathematical Institute
ul. Vavilova 42
117966 Moscow GSP-1
Russia