

## HOMEWORK 4 (18.314, FALL 2006)

1. Problem 4.1.3 (p. 143) from the book (MN).
2. Problem 4.1.5 (p. 143)
3. Problem 7.5.4 (p. 239)
4. a) Problem 7.1.2 (p. 224).    b) Problem 7.5.3 (p. 239)
5. Problem 4.2.2 (p. 150)
6. Problem 7.5.2 (p. 238). Use only basic results on trees to prove the recurrence relation in a). It's ok to compute the determinant on a computer for c).
7. Consider a graph  $G_n$  on  $\{1, \dots, n\}$  with edges:  $(i, i + 1)$ ,  $1 \leq i \leq n - 1$  and  $(j, j + 2)$ ,  $1 \leq j \leq n - 2$ .
  - a) Compute the  $t(G_n)$  for  $2 \leq n \leq 6$  (you can use a direct calculation, the matrix tree theorem, or a recurrence relation - anything will do)
  - b) Guess the answer based on the first values (to help you guess you can use the **integer sequences** web site - google this). Find a recurrence relation these numbers satisfy.
  - c) Prove your guess in part b).
8. Use the matrix tree theorem to find a polynomial time algorithm for constructing a random spanning tree in a graph. (Hint: for a given edge  $e$  first compute probability that this edge is in the tree. Then use recursion.)

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This Homework is due Wednesday October 18 at 14:05 am.