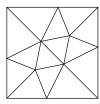
## HOMEWORK 1 (18.315, FALL 2003)

- **Def.** A proper coloring of a graph is a coloring of vertices with no monochromatic edges. A grid graph  $G_{m,n}$  is a product of a m-path and a n-path.
- 1) Let c(n) be the number of proper colorings of  $G_{n,n}$  with 3 colors. Prove a)  $c(n) > C (1+\varepsilon)^{n^2}$  for some  $C, \varepsilon > 0$ ; b)  $\frac{\log c(n)}{n^2} \to \alpha$  as  $n \to \infty$ , for some  $\alpha > 0$ .
- 2) Denote by  $N_k$  be the number of proper colorings of  $G_{n,n}$  with k colors. Approximate  $N_k$ up to 10% when
  - a) n = 100 and k = 1,000,000;
  - b) n = 100 and k = 1,000.
- 3) Consider the set  $S_k(n)$  of proper colorings of  $G_{n,n}$  with k colors. Prove that for every two colorings  $\chi, \chi' \in \mathcal{S}_k(n)$ , one can go from  $\chi$  to  $\chi'$  by changing one color at a time, when
  - a) k = 5;
  - b) k = 4.
- 4) In Schur's theorem, the proof we presented gives n(r) < er!. Find an exponential lower bound by an explicit construction.
- 5) Consider random graphs H on n vertices with m=2n edges (defined as subgraphs of a complete graph  $K_n$ ). What is more likely: that H is bipartite or not?
- 6) An acute decomposition of a polygon  $P \subset \mathbb{R}^2$  is a subdivision of P into acute triangles, such that there are no vertices lying on the interior edges (see Figure 1). Prove that an acute decomposition of P always exist if
  - a) P is a triangle;
  - b) P is a convex polygon which has an inscribed circle;
  - c) P is any convex polygon.



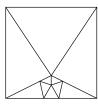


FIGURE 1. A valid acute decomposition of a square, and an invalid one.

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This Homework is due on Wednesday Sep 21 at 4 pm. in my office (2-390) or by e-mail. Please remember to write the name(s) of your collaborators (see collaboration policy).