MIDTERM 2 (MATH 61, SPRING 2017)

Your Name: ____

UCLA id: ____

Math 61 Section: ___________

Date: ______________________

The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no phones, no web access. You MUST write your name and UCLA id. Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught violating the rules get automatic 10% score deduction.

Points:

1	
2	
3	
4	
5	

____

Total: **(out of 100)**

Problem 1. (15 points)

Solve the following LHRR: $a_{n+1} = 5a_n - 6a_{n-1}$, $a_1 = 9$, $a_2 = 20$.

Solution:

characteristic polynomial: $x^2 = 5x - 6 \Rightarrow x_1 = 2, x_2 = 3$
We have: $a_n = A \cdot 2^n + B \cdot 3^n$
Plug in a_1 and a_2 we get $A = \frac{7}{2}, B = \frac{2}{3}$

Problem 2. (20 points)

Find the number of subgraphs of G isomorphic to H, where

a) $G = C_{12}$, $H = P_5$.
Answer: 12

b) $G = K_{9,9}$, $H = C_6$.
Answer: $\binom{9}{3}^2 \times 6$
Hint: First choose $K_{3,3}$, then there are 6 C_6 in it

c) $G = K_{9,9}$, $H = C_7$.
Answer: 0

d) $G = H_3$ (the 3-cube), $H = C_4$.
Answer: 6 (only the 6 faces of the cube count)

e) $G = H_3$ (the 3-cube), $H = P_4$.
Answer: 48 (Fix the central edge of P_4: There are 12 different central edges for P_4, each of them could induce 4 P_4)

Problem 3. (20 points)

For each of these sequences, either draw a simple graph with this score (degree sequence), or explain why there is no such graph.

a) (3,3,3,1,1,1)

Exist: Just draw a triangle and 3 extra vertices (one to one) connecting to each vertex of the triangle.

b) (4,4,4,1,1,1)

Not Exist: sum of degree is odd.

c) (5,5,5,3,2,1)

Not Exist: sum of degree is odd.

d) (4,4,...,4) ← 16 numbers.

Exist: You can draw 2 copies of $K_{4,4}$

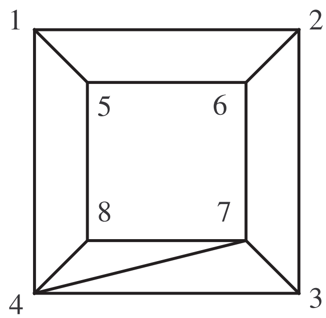
e) (3,3,3,3,2,2,2,2)

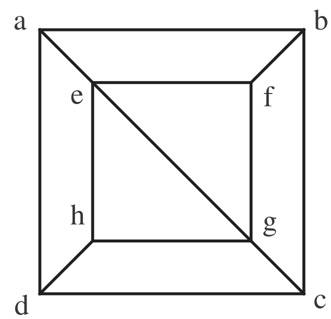
Exist: You can draw a C_8 and add 2 chords

Problem 4. (15 points)

Decide whether the following pairs of graphs on 8 vertices are isomorphic or non-isomorphic.

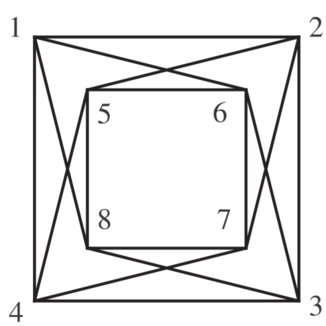
a)

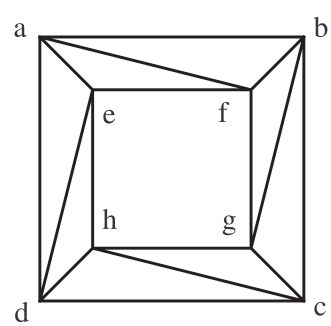




Isomorphic:, for example: {1,2,3,4,5,6,7,8}→{a,b,f,e,d,c,g,h}
You need to make sure that if 4 vertices construct a C_4 then their mapped vertices also form a C_4.

b)





Not isomorphic: the left graph does not have C_3 as subgraph but the right one does.

Important: In case of isomorphism, you must present a bijection. In case non-isomorphism, you must present an argument.

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T **F** (1) The hypercube graph H_4 contains an Eulerian cycle.

T **F** (2) The hypercube graph H_4 contains a Hamiltonian cycle.

T **F** (3) A subgraph of a connected graph is always connected.

T **F** (4) A subgraph of a disconnected graph is always disconnected.

T **F** (5) The sum of degrees of $K_{\ell,\ell}$ is ℓ^2.

T **F** (6) The sum of degrees of a graph on $n \geq 3$ vertices is smaller than $n^2 - 1$.

T **F** (7) Computing the number of walks of given length in a graph can be done efficiently.

T **F** (8) Deciding whether a graph has Hamiltonian cycle can be done efficiently.

T **F** (9) The number of (shortest) grid walks $(0,0) \to (5,5)$ which do not go through $(1,3)$, $(3,4)$, $(4,1)$, $(1,5)$ is > 100.

T **F** (10) Graph H is a subgraph of G. Graph H contains a Hamiltonian cycle. Then G contains a Hamiltonian cycle.

T **F** (11) Graph H is a subgraph of G. Graph G contains a Hamiltonian cycle. Then H contains a Hamiltonian cycle.

T **F** (12) Graph $K_{50,50}$ contains a subgraph isomorphic to K_{10}.

T **F** (13) Graph $K_{50,50}$ contains a subgraph isomorphic to $K_{10,10}$.

T **F** (14) The number of walks $1 \to 1$ of length k in a graph G on n vertices can be computed via matrix $(A_G)^n$.

T **F** (15) Isomorphic graphs have the same number of Eulerian circuits.

TTFFF TTFFF FFTFT