

HOMEWORK 4 (MATH 61, SPRING 2017)

Read: **RJ**, Sec. 6.3 and first half of 6.7 (before identities), Sec. 7.1, 7.2, Sec 8.1.

Solve: **RJ**, Sec. 6.3 Ex 2, 3, Sec. 6.7 Ex 15, 20, Sec 7.2 Ex. 16, 17, 22, Sec 8.1 Ex 9, 10.

I. Find the number of anagrams of MISSISSIPPI which:

- a) begin with I,
- b) begin and end with S,
- c) have two P's in a row,
- d) have four S's in a row,
- e) have second and fourth letter S,
- f) have all P's before I's,
- g) have S's and I's alternate (beginning with I),
- h) have first to fifth letters all different,
- i) begin or end with a P,
- j) begin or end with a S.

II. On a grid, let $A = (0, 0)$, $B = (10, 12)$, $C = (3, 2)$, $D = (6, 9)$, $E = (9, 7)$. Find the number of (shortest) grid walks $A \rightarrow B$ which

- a) go through C
- b) do not go through D
- c) go through C but not D
- d) go through E but not C
- e) go through C and E
- f) go through either D or E
- g) go through D and E
- h) go through neither C nor D nor E .

III. Prove the following results about Fibonacci numbers F_n :

- a) there are infinitely many n such that $F_n = 0 \pmod{7}$,
- b) there are infinitely many n such that F_n begins with 2.

IV. Draw two non-isomorphic graphs with scores (degree sequences)

- a) $(3, 3, 3, 3, 5, 5, 6, 6, 6)$
- b) $(3, 3, 3, 5, 5, 5, 6, 7, 7)$

V. Let $a_n = F_1 + F_3 + \dots + F_{2n-1}$, where $\{F_i\}$ are the Fibonacci numbers. Guess the formula for a_n and prove it by induction.

VI. Use a closed formula for the Fibonacci numbers (in terms of powers of the golden ratio), to prove:

$$F_{n+1} F_{n-1} = F_n^2 + (-1)^n.$$

VII. Solve LHR $a_{n+1} = 4a_n - 3a_{n-1}$ for the following sets of initial conditions:

(i) $a_1 = 1, a_2 = 3,$

(ii) $a_1 = 5, a_2 = 5,$

(iii) $a_1 = 2, a_2 = 4.$

Use induction to verify your formulas in all three cases.

This Homework is due Wednesday May 10, at 10:59:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Write the answers in inc and box them. Remember that in the proof questions, you also need to provide an explanation exhibiting your logic. In other questions, just the answer suffices.

P.S. Each item above has the same weight.