HOMEWORK 3 (MATH 61, SPRING 2017)

Read: **RJ**, Sec. 6.2 and first half of 6.7 (before identities).

Solve: **RJ**, Sec. 6.2 Ex 6, 8, 29, 34, 35, 37, Sec. 6.7 Ex 2, 4, 5.

I. Throughout the problem, assume $n = 12$. Compute the number of permutations $(a_1, \ldots, a_n)$ of $\{1, 2, \ldots, n\}$, such that

a) $a_1 \cdot a_n = 6$
b) $a_1 - a_n = n - 1$
c) $a_1 + a_n = n + 2$
d) $a_1 = 1$ and $a_n = n$
e) $a_1 = 2$ or $a_2 = 3$
f) $a_1 \leq 3$ or $a_2 \geq 3$
g) $a_1 = 2$, or $a_2 = 3$, or $a_3 = 4$

II. Throughout the problem, assume $n = 12$ and $k = 4$. Compute the number of k-subsets A of $[n]$, such that:

a) A contains 1 and n
b) A contains 1 but not n
c) A contains 1 or n
d) A contains at least one integer ≤ 5
e) A contains at least one integer ≤ 3 and is missing at least one integer ≥ 10
f) A contains at least two integers ≤ 6
g) A contains only even integers

III. Let $X = \{0, 1, 2, \ldots, 11\}$. For each of these functions $f : X \to X$, decide whether they are a injection, surjection, bijection, or neither.

a) $f(x) := x + 1 \mod 12$
b) $f(x) := 5x \mod 12$
c) $f(x) := x^2 \mod 12$
d) $f(x) := x^3 \mod 12$
e) $f(x) := 5^x \mod 12$
f) $f(0) = 0$, $f(x + 1) = 2f(x) + 1 \mod 12$, for $x = 0, \ldots, 10$.
g) $f(x) = \binom{2x+3}{x+1} \mod 12$

Note: The way to read this is to use modular arithmetic. For example, in *b*), the function f maps $0 \to 0$, $1 \to 5$, $2 \to 10$, $3 \to 3$ (=15 mod 12), $4 \to 8$ (=20 mod 12), etc.

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This Homework is due Wednesday April 26, at 10:59:59 am (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Write the answers in inc and box them. Remember that in the proof questions, you also need to provide an explanation exhibiting your logic. In other questions, just the answer suffices.

P.S. Each item above has the same weight.