MIDTERM 1 (MATH 61, SPRING 2015)

Your Name: ______________________________

UCLA id: ______________________________

Math 61 Section: ________

Date: ______________

The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no web access. You MUST write your name and UCLA id.
Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught writing after time get automatic 10% score deduction.

Points:

1	
2	
3	
4	
5	

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Total: **(out of 100)**

Problem 1. (20 points)

Compute the number of permutations $(x_1, x_2, \ldots, x_n)$ of $\{1, 2, \ldots, 9\}$ such that:

a) $x_1 = 2$,

b) $x_1 \cdot x_2 \cdot x_3 = 6$,

c) $x_1 = x_2 = x_3 \text{ mod } 7$,

d) $x_1 < x_2 < 5$.

Problem 2. (20 points)

Let $X = \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ be the set of all integers. For each of these relations R, decide whether they are reflexive, symmetric or transitive (or neither).

a) xRy if and only if $|x| = |y|$.

b) xRy if and only if $x + 2y = 0 \bmod 3$.

c) xRy if and only if $x^2 + 2y^2 = 0 \bmod 3$.

d) xRy if and only if $x^3 + 122y^3 = 0 \bmod 3$.

Problem 3. (15 points)

Let $A = (0, 0)$, $B = (10, 10)$. Find the number of (shortest) grid walks γ from A to B, such that:

a) γ never visits points $(0, 10)$, $(10, 1)$, $(5, 5)$.

b) γ visits all points $(1, 1)$, $(2, 2)$, $(3, 3)$, $\ldots$, $(9, 9)$.

c) γ visits points $(5, 0)$ and $(5, 10)$, but not $(5, 5)$.

Problem 4. (15 points)

Recall the Fibonacci sequence: $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, etc.

Prove that $F_n \leq 2^{n-1}$.

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T **F** (1) The number of functions from $\{A, B, C, D\}$ to $\{1, 2, 3\}$ is equal to 4^3.

T **F** (2) The sequence $10, 21, 32, 43, \ldots$ is increasing.

T **F** (3) The sequence 2/1, 3/2, 4/3, 5/4 is non-increasing.

T **F** (4) There are 20 anagrams of the word $BUBUB$.

T **F** (5) There are more anagrams of the words $AAAACCC$ which begin with A than with C.

T **F** (6) There are infinitely many Fibonacci numbers $= 1 \bmod 3$.

T **F** (7) There are infinitely many binomial coefficients $\binom{n}{k} = 1 \bmod 17$.

T **F** (8) Each of the 14 students wrote on a paper 10 distinct numbers, from the set $\{1, 2, \ldots, 100\}$. Then there are two students who have at least 2 numbers in common on their lists.

T **F** (9) The probability that a random 10-subset of $\{1, 2, \ldots, 19\}$ contains 10 is equal to $1/2$.

T **F** (10) For every two subsets $A, B \subset U$, we must have $|A \smallsetminus B| = |B \smallsetminus A|$.

T **F** (11) For every two subsets $A, B \subset U$, we must have $|A \cup B| \geq |\overline{B}|$

T **F** (12) Every surjection that is also a bijection must be also an injection.

T **F** (13) Every surjection that is also an injection must be also a bijection.

T **F** (14) Let $\mathcal{A}$ be the set of 3-subsets of $[9] = \{1, 2, \ldots, 9\}$. Similarly, let $\mathcal{B}$ be the set of 6-subsets of [9]. Consider a map $f : A \to [9] \smallsetminus A$. Then f is a bijection from $\mathcal{A}$ to $\mathcal{B}$.

T **F** (15) The pigeon hole principle was proved in class by induction.