Warning: everywhere below we follow book notation:

\[ P(n, k) = \frac{n!}{(n-k)!} \quad \text{and} \quad C(n, k) = \binom{n}{k}. \]

6.2
6. \( P(11, 5) = \frac{11!}{6!} \).
8. \( P(12, 4) = \frac{12!}{8!} \).
29. \( C(12, 4) = \frac{12!}{8!4!} \).
34. \( C(6, 3) \cdot C(7, 4) \).
35. \( \# \text{ of Total Committees} - \# \text{ of All Male Committees} = C(13, 4) - C(6, 4). \)
37. \( \# \text{ of Total Committees} - \# \text{ of All Male Committees} - \# \text{ of All Female Committees} = C(13, 4) - C(6, 4) - C(7, 4). \)

36.7
2. \( (2c - 3d)^5 = \sum_{k=0}^{5} C(5, k)2^k(-3)^{5-k} \)
4. \( C(6, 6)2^6(-1)^6 = \frac{12!2^6}{6!6!} \)
5. \( C(10, 5)C(5, 3) = \frac{10!}{5!2!3!} \)

I.
a) \( a_1a_n = 4 \) means that either \( a_1 = 1 \) and \( a_n = 4 \), or \( a_1 = 4 \) and \( a_n = 1 \). In both cases we can assign the remaining \( n-2 \) values for \( a_2, \ldots, a_{n-1} \) arbitrarily, so there are a total of \( 2 \cdot (n - 2)! \) permutations with \( a_1a_n = 4 \). Answer: \( 2 \cdot 9! \).

b) We have \( a_1 - a_n = n - 1 \) if and only if \( a_1 = n, a_n = 1 \). There are \( 9! \) ways to order the other 9. Answer: \( 9! \).

c) \( a_1 \) can be \( 2, 3, \ldots, n \) and then \( a_n \) is determined (it must be \( n + 2 - a_1 \)). Notice that \( a_1 \neq (n + 2) - a_1 \) since \( n + 2 = 13 \) so \( a_1 \) is even if and only if \( (n + 2) - a_1 \) is odd. For each of the 10 choices for \( a_1 \), there are \( 9! \) ways to order the other 9. Answer: \( 10 \cdot 9! = 10! \).

d) \( (n - 2)! \). Answer: \( 9! \).

e) Number of permutations with \( a_1 = 2 \) is \((n - 1)\)!; number of permutations with \( a_2 = 2 \) is \((n - 1)\)! Since they cannot both happen at the same time, the number of permutations when at least one of them is true is \( 2 \cdot (n - 1)! \). Answer: \( 2 \cdot 10! \).

f) Number of permutations with \( a_1 = 2 \) is \((n - 1)\)!; number of permutations with \( a_2 = 3 \) is \((n - 1)\)!; number of permutations when both happen is \((n - 2)\)!; so the total is \( 2 \cdot (n - 1)! - (n - 2)! \) by the inclusion-exclusion principle. Answer: \( 2 \cdot 10! - 9! \).

g) The inclusion-exclusion principle with three sets says that \( |X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z| \). Using this we get that the number of permutations here is \( 3 \cdot (n - 1)! - 3 \cdot (n - 2)! + (n - 3)! \). Answer: \( 3 \cdot 10! - 3 \cdot 9! + 8! \)

II.
a) We need \( k - 2 \) other elements from \( \{2, \ldots, n - 1\} \). There are \( C(n - 2, k - 2) \) ways to choose them. Answer: \( C(9, 2) \).
b) We need \(k - 1\) elements from \(\{2, \ldots, n - 1\}\). There are \(C(n - 2)(k - 1)\) ways to choose them. Answer: \(C(9, 3)\).

c) By the inclusion-exclusion principle, \(C(n - 1)(k - 1) + C(n - 1)(k - 1) - C(n - 2)(k - 2)\). Answer: \(2 \cdot C(10, 3) - C(9, 2)\).

d) There are \(C(n - 4, k)\) choices which do not contain at least one integer \(\leq 4\). Answer: \(C(11, 4) - C(7, 4)\).

e) \(A\) has only four elements, so it is always missing at least one integer \(\leq 6\) (pigeonhole principle). So we can forget about the second condition. Similarly to \(d\), we get \(C(n, k) - C(n - 3, k)\). Answer: \(C(11, 4) - C(8, 4)\).

f) We subtract subsets with fewer than \(2\) numbers less than or equal to \(6\) from the total number of \(k\)-subsets to get \(C(n, k) - C(n - 6, k) \cdot C(6, 0) - C(n - 6, k - 1) \cdot C(6, 1)\). Answer: \(C(11, 4) - C(5, 4) - 6 \cdot C(5, 3)\).

g) We choose \(k\) numbers from the \(5\) evens in \([n]\). Answer: \(C(5, 4)\).

III.

Because \(f\) maps a finite set to itself, injection, surjection, and bijection are all the same.

a) It is injective because if \(x + 1 = y + 1 \mod 10\), then \(x = y \mod 10\) and each element of \(X\) is in a different equivalence class \(\mod 10\). Answer: Bijection.

b) It is injective because if \(3x = 3y \mod 10\), then \(10\) divides \(3(x - y)\). Since \(2\) and \(5\) are prime and they don’t divide \(3\), they must both divide \(x - y\). Thus \(x - y\) is divisible by \(10\) so \(x = y \mod 10\). Answer: Bijection.

c) It is not injective because \(1^2 = 9^2 \mod 10\). Answer: Neither.

d) The easiest way to see that it is injective is just to calculate all the values. Answer: Bijective.

e) Except for \(x = 0\), \(3^x\) is always odd so it can only lie in \([1], [3], [5], [7]\), or \([9]\) so it cannot be surjective. Answer: Neither.

f) Except for when \(x = 0\), \(f(x) = 2c + 1\) for some integer \(c\) so, in particular, it must be odd. Therefore \(f\) cannot be surjective as in part (e). Answer: Neither.

g) It’s not injective because \(f(3) = 6 = f(5)\). Answer: Neither.