Problem I. Find the number of grid walk from $(0, 0)$ to $(10, 10)$ which never enter the diagonal \{(1, 1), (2, 2), \ldots, (9, 9)\}. Recall that grid walks can go only Up and Right.

Problem II. Find the number of grid walk from $(0, 0)$ to $(10, 10)$ which always stay on or above diagonal, and which go through $(4, 4)$.

Problem III. Find the number of grid walk from $(0, 0)$ to $(10, 10)$ which always stay on or above diagonal, and which go through $(4, 4)$ and $(7, 7)$.

Problem IV. Find the number of grid walk from $(0, 0)$ to $(10, 10)$ which always stay on or above diagonal, and which go through $(4, 4)$ or $(7, 7)$.

Problem V. Find the number of triangulations of 10-gon with vertices \{1, 2, \ldots, 10\}, which contain diagonal (1, 5).

Problem VI. Find the number of triangulations of 10-gon with vertices \{1, 2, \ldots, 10\}, which contain triangle (1, 5, 8).

Problem VII. Find the number of triangulations of 10-gon with vertices \{1, 2, \ldots, 10\}, which do not contain diagonals (1, 5) and (1, 7).

Problem VIII. Let $a_n = F_1 + F_3 + \ldots + F_{2n-1}$, where \{F_i\} are Fibonacci numbers. Guess the formula for $a_n$ and prove it by induction.

Problem IX. Prove the following formula:

$$F_{n+1}F_{n-1} = F_n^2 + (-1)^n$$

Use either induction or a closed formula for Fibonacci numbers.

Problem X. Solve LHRR $a_{n+1} = 4a_n - 3a_{n-1}$ for the following sets of initial conditions:

(i) $a_1 = 1$, $a_2 = 3$,
(ii) $a_1 = 5$, $a_2 = 5$,
(iii) $a_1 = 2$, $a_2 = 4$.

Use induction to verify your formulas in all three cases.

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This Homework is due Wednesday Nov 6, at 12:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators’ names at the end. Write the answers in inc and box them. Remember that you also need to provide an explanation exhibiting your logic.

P.S. Each item above has the same weight.