I. Find the number of anagrams of MISSISSIPPI which:
   a) begin with M,
   b) begin and end with S,
   c) have two P's in a row,
   d) have four S's in a row,
   e) have second and fourth letter I,
   f) have all I's before P's,
   g) have S's and I's alternate (beginning with S),
   h) have first fourth letters all different,
   i) begin or end with a P,
   j) begin or end with a S.

II. Prove the following results about Fibonacci numbers $F_n$:
   a) there are infinitely many $n$ such that $F_n = 0 \mod 5$,
   b) there are infinitely many $n$ such that $F_n$ begins with 1.

III. Use variations on the Pigeonhole Principle to prove the following results:
   a) There exists a number $N = 33\ldots 3$, such that $97 | N$. Prove that such $N$ can be found with fewer than 100 digits.
   b) Botanic Garden has trees of many types. It is known that there are 9999 trees in the Botanic Garden. Prove that either there are at least 100 types of trees, or there are at least 100 trees of the same type.
   c) There are 66 points in the unit square $[1 \times 1]$. Prove that at least two of them are at distance $< 0.2$.
   d) There are 10 people in the class. At the end of the midterm, each sent an email to exactly 5 of them. Prove that there are two people who sent emails to each other.

This Homework is due Wednesday Oct 23, at 12:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators’ names at the end. Write the answers in inc and box them. Remember that in the proof questions, you also need to provide an explanation exhibiting your logic. In other questions, just the answer suffices.

P.S. Each item above has the same weight.