MIDTERM PRACTICE PROBLEMS
(MATH 4653, FALL 2008)

1. A fair coin is flipped 9 times. Let x be the number of Heads and let y be the number of Tails. Compute $P(x=0)$, $P(x=1)$, $P(x<y)$, $P(x^2+y^2=10)$.

2. Let $(a_1,\ldots,a_8)$ be a random permutation of $\{1,\ldots,8\}$. Compute:
a) $P(a_1+a_2=8)$, $P(a_1\cdot a_2=2)$,
b) $P(a_1+a_2=8 \mid a_1\cdot a_2=2)$, $P(a_1+a_2=8 \mid a_3\cdot a_4=2)$,

3. Let a,b be two numbers chosen independently at random from $\{1,\ldots,10\}$. Which pairs of the following events are independent? (you need to check all 3 pairs)

$A=\{a\le 5\}$, $B=\{a\ne b\}$, $C=\{a+b\le 6\}$.

4. Let X be a random integer chosen from a non-uniform distribution on $\{0,1,\ldots,29\}$, such that $P(X=n)=p$ if the integer n has no digits 3, and $P(X=n)=2p$ otherwise. Find p. Compute $P(X<10)$, $P(X<5)$ and $E[X]$. How would $E[X]$ change if 3 is replaced by 7?

5. Suppose $P(A\cup B)=0.7$, $P(A\mid B)=0.2$, and $P(B\mid A)=1/3$. Compute $P(A)$.

6. Let X be a discrete random variable with $P(X=1)=\frac{1}{2}$, $P(X=2)=\frac{1}{3}$ and $P(X=3)=\frac{1}{6}$.

a. Compute $E[X]$ and $Var(X)$.

b. Compute $E[Y]$ and $Var(Y)$, where $Y=2-3\,X$.

7. Three fair dies are thrown simultaneously. If all three have even numbers on them, we stop. If not, we repeat (throw all three dies again). Denote by X the number of throws until we get all even numbers.

a. Compute $P(2\le X\le 5)$.

b. Compute $E[X]$ and $Var(X)$.

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8. True or False? (give only answers, no calculations are necessary)

a. In the coupon collector's problem with 50 coupons, the expected time to collect the first 25 coupons is more than half the expected time to collect all 50 coupons.

b. For all events A, B we have $P(A \cup B) \geq 1 - P(A \cap B)$.

c. For every X with $\mathrm{Poi}(\lambda)$ distribution, we have $P(X \geq 4\lambda) \leq 0.25$.

d. Every two Bernoulli distributions are independent.

e. Every Bernoulli distribution is also a Binomial distribution.

f. The variance of a geometric distribution is smaller than the square of the mean.

g. In the princess marriage problem with 100 princes the odds of choosing the best prince is the second strategy of following two strategies is better:

1. skip the first prince and choose the first prince who is better than at least one of the previous princes.
2. skip the first 50 and choose the first prince who is better than at least one of the previous princes.

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Bernoulli distribution $\mathrm{Ber}(p)$
$P(X = 1) = p$, $P(X = 0) = 1 - p$, $E[X] = p$, $\mathrm{Var}(X) = p(1 - p)$.

Binomial distribution $\mathrm{Bin}(n, p)$
$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$, $E[X] = np$, $\mathrm{Var}(X) = np(1 - p)$.

Geometric distribution $\mathrm{Geo}(p)$
$P(X = k) = p(1 - p)^{k-1}$, $E[X] = 1/p$, $\mathrm{Var}(X) = (1 - p)/p^2$.

Poisson distribution $\mathrm{Poi}(\lambda)$
$P(X = k) = e^{-\lambda} \lambda^k / k!$, $E[X] = \lambda$, $\mathrm{Var}(X) = \lambda$.