FINAL PREPARATION PROBLEMS (4653, FALL 2008)

I. Let $f(x,y) = c(x^2/y)$ for all $1 \le x, y \le 2$ and $f(x,y) = 0$ otherwise.
$a)$ Compute constant c and the correlation coefficient.
$b)$ Decide whether X, Y are independent.
$c)$ Find the cumulative probability function for X.

II. Denote by A a region in the plane defined by the following inequalities: $1 \le x, y \le 3$, $y - x \le 2$, $x + y \le 4$. Let (X, Y) be coordinates of a random point in A.
$a)$ Compute $P(X \le 2)$, $P(Y \le 2)$, $P(X + Y \ge 1)$, $P(X^2 + Y^2 \le 2)$.
$b)$ Compute $P(X \le 2 \mid Y \ge 2)$, $P(X + Y \ge 1 \mid Y \le 2)$, $P(X^2 + Y^2 \le 2 \mid X \le 1)$.
$c)$ Find $\rho(X, Y)$.

III. Suppose X is Poly(3) c.r.v, and $Y = X^2 - 1$. Find $\rho(X, Y)$.

IV. Let X be a c.r.v. with p.d.f. $f(x) = c(x - x^3)$ for $0 \le x \le 1$ and $f(x) = 0$ otherwise. Find c.
$a)$ Compute $P(0.1 \le X \le 0.3)$, $P(X \ge 0.8)$,
$b)$ Compute $P(X^2 \le 0.25)$, $P(X^2 = \frac{1}{2})$, $P(e^X > 3)$.
$c)$ Compute $E[X]$ and $\mathrm{Var}(X)$.
$d)$ Use Chebyshev inequality to estimate $P(X > 0.8)$. Find by how many % off is this estimate.

V. Let R be a circle of radius 2 around point $(1, 0)$. Let $a = (X, Y)$ be a random point in R, and let L be the distance from a to the origin $(0, 0)$.
$a)$ Compute $P(L \le 1)$, $P(L \le 2)$, $P(L \le 3)$.
$b)$ Compute $P(L \le 1 \mid X \ge 0)$, $P(L \le 1 \mid X \le 1)$.
$c)$ Compute p.d.f. and c.p.f. of L.
$d)$ Find $E[L]$ and $\mathrm{Var}(L)$.

VI. True or False?

$a)$ If X is Exp(2) c.r.v., then $2X$ is Exp(4) c.r.v.

$b)$ If X is Poly(2) c.r.v., then X^2 is Poly(4) c.r.v.

$c)$ Suppose X is a c.r.v. such that its p.d.f. $f(x) > 0$ only for $0 \le x \le 1$. Then $E[X] < 2$.

$d)$ Suppose X and Y are identical and independent c.r.v. Then $E[X + Y] \ge E[X]$.

e) Suppose X and Y are identical and independent c.r.v. Then $\mathrm{Var}(X+Y) \geq \mathrm{Var}(X)$.

f) If X and Y are independent, then $f(x, y) = f_X(x) + f_Y(y)$ for all x, y, where f_X and f_Y are marginal p.d.f.'s

g) If X and Y are c.r.v. with distributions Exp(1) and Exp(2), respectively, then they cannot be independent.

h) If X has a $U(10, 20)$ distribution, then $X/10$ has $U(1, 2)$ distribution.

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GOOD LUCK!