HOMEWORK 1 (MATH 184, WINTER 2018)

Read: Bona (Second ed.), sections 1.1-4, 2.3-4, 3.1-2, 4.1 and 4.4.

Practice Problems: Exc 3, 4, 17a, 35, 37, 45, 46, in §1.10, 18, 19, 20, 22, 23, 25, 26, 28, 29, 30 in §2.10, 11, 12 in §3.10, 13, 14, 23, 25 in §4.10,

Solve these problems:

I. Consider a jar with 30 red, 30 green, 30 yellow and 90 brown m&m chocolates. Choose randomly 30 of them. Compute:

a) probability that no red are chosen
b) probability all four colors are present
c) probability there are exactly the same number of red as green
d) probability there are exactly the same number of red as black
e) expected number of red
f) expected (number of brown) minus (number of red)
g) expected number of colors chosen
h) expected number of purple
i) probability there are even number of each color
j) probability there are prime number of each color

II. Let $\sigma \in S_{20}$ is a random permutation. Compute:

a) probability that σ has exactly one fixed point
b) probability that σ has only cycle divisible by 5
c) probability that σ has cycles of distinct lengths
d) probability that the cycle containing 1 has length a prime number
e) probability that the cycle containing 1 also contains 2
f) probability that the cycle containing 1 also contains 2 and 3
g) probability that the cycle containing 1 also contains 2 or 3
h) the expected number of prime numbers contained in a cycle containing 1
i) the expected number of i such that $\sigma(i)$ has the same parity
j) the expected number of i such that $|\sigma(i) - i| \geq 3$

III. Recall Stirling numbers of the first an second kind. We proved in class that $S(n, 2) = 2^{n-1} - 1$ and $c(n, 1) = (n - 1)!$.

a) Obtain a similar explicit formula for $S(n, 6)$.
b) Obtain a similar explicit formula for $S(n, n - 2)$.
c) Obtain a similar explicit formula for $c(n, 3)$.
d) Obtain a similar explicit formula for $c(n, n - 3)$.

IV. For the following six functions decide which asymptotics formulas of the type $f = o(g)$ hold:

$$a(n) = n!\,, \quad b(n) = n^{\sqrt{n}}\,, \quad c(n) = (n/2)^n\,, \quad d(n) = (\log n)^{n/\log n}$$

$$u(n) = 2^{n^{\sqrt{2}+\sin n}}\,, \quad v(n) = \big(\sqrt{n}(\log n)\big)^{\sqrt{n}(\log n)}$$

You can use Stirling's formula for $n!$

V. Let $n = 100$. Recall the n hunters shooting n rabbits problem. Assume the rabbits stand in a circle. Compute:
a) the expected number of rabbits shot by 2 or more hunters
b) the probability exactly two rabbits survive, and they stand next to each other
c) the expected number of surviving rabbits whose both neighbors survive
d) the probability that no rabbit is shot by > 50 hunters

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This Homework is due Friday February 9, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.