**HOMEWORK 3 (MATH 180, SPRING 2016)**

**Read:** **MN** (Second ed.), sections 4.5, 4.7

**Solve:** Exc 9a in §4.4, 6, 8 in §4.5 in and the following problems.

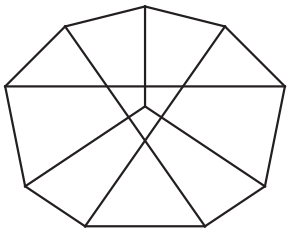
**I.** Let $G = (V, E)$ be a graph on $n = |V|$ vertices. Suppose the distance $d_G(v, w) = n - 1$ for some vertices $v, w \in V$. Prove that $G \simeq P_n$ and that $v, w$ are endpoints.

**II.** Find all connected graphs on 6 vertices, up to isomorphism, where the distances between every two vertices is at most 2.

**III.** Prove that $K_{p,q}$ does not have a Hamiltonian cycle, for all $p \neq q$.

**IV.** Find the smallest complete bipartite graph $G$ which contains $C_{12}$ and $K_{8,4}$ as its subgraphs. Same question without requirement that $G$ is complete bipartite.

**V.** Prove that the following graph on 10 vertices does not have a Hamiltonian cycle.



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This Homework is due Wednesday April 20, at 12:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. Box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic.