HOMEWORK 6 (MATH 180, SPRING 2014)

Read: **MN**, sections 6.1-4

Solve:

I. A graph G is called *cubic* if every vertex of G has degree 3.

a) Find two non-isomorphic cubic graphs whose faces are either quadrilaterals or hexagons.

b) Use Euler's formula to show that all such graphs must have exactly 6 quadrilateral faces.

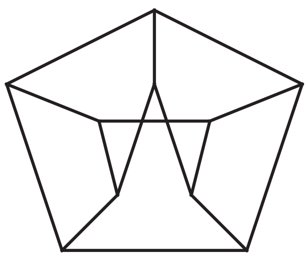
II. Suppose graph G on n vertices has N proper 4-colorings. Prove that G has at least $2^n \cdot N$ proper 8-colorings.

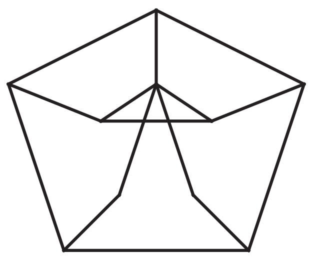
III. Suppose planar graph G has a Hamiltonian cycle. Prove that the dual graph G^* is 4-colorable.

IV. Let P be a convex polytope whose vertices have degrees ≥ 4. Prove that it has at least eight triangular faces.

V. Let G be a connected graph with $n \geq 4$ vertices and m edges. Prove that G is planar if $m = n$. Same for $m = n + 1$ or $m = n + 2$.

VI. Decide whether following graphs are planar. You can use Kuratowski's theorem (thm 6.2.4) or a direct argument.





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This Homework is due Friday June 6, at 12:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end. You MUST box all answers. Remember that answers are not enough, you also need to provide an explanation exhibiting your logic.