

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**Instructions:**

- There are 6 problems. Make sure you are not missing any pages.
- Show all work in detail or your answer will not receive credit.
- Write neatly and box all answers.
- No calculators, books, or notes are allowed.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- Do not use your own scratch paper. Extra scratch paper is available from the front of the room.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	10	
6	10	
<b>Total:</b>	<b>100</b>	

1. (20 points) Solve the initial value problem:

$$t \frac{dy}{dt} + (2t^2 + 3t^3)y = te^{-(t^2+t^3)}, \quad y(0) = 5$$

2. (a) (10 points) Solve the initial value problem:

$$\frac{dy}{dt} = 2t(y - 1)^2, \quad y(0) = 0$$

(b) (10 points) Solve the initial value problem:

$$\frac{dy}{dt} = 2t(y - 1)^2, \quad y(0) = 1$$

3. (a) (10 points) Show that the following equation is exact:

$$2x + y^2 + 2xy \frac{dy}{dx} = 0$$

- (b) (10 points) Find the explicit solution to the initial value problem:

$$2x + y^2 + 2xy \frac{dy}{dx} = 0, \quad y(1) = 1$$

4. (20 points) Find an integrating factor for the following equation (you do not need to solve the equation):

$$4x^2y + 2y^2 + (3x^3 + 4xy)\frac{dy}{dx} = 0$$

*Hint:* It is of the form  $\mu(x, y) = xy^a$  for some  $a$ .

5. (10 points) Solve the initial value problem:

$$\frac{dy}{dx} = \sin(\pi y)(2^y - 4)(y^3 - 6y^2 + 12y - 8), \quad y(1) = 2$$

6. (10 points) Find the equilibrium solutions for the following equation, and identify them as asymptotically stable or unstable.

$$\frac{dy}{dt} = y^2 - 5y + 6$$

Extra Scratch Paper: