

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Instructions:

- There are 7 problems, one of which has 3 parts. Make sure that you are not missing any pages.
- Show all work in detail or your answer will not receive credit.
- Write neatly and box all answers.
- No calculators, PDA's, cell-phones, books, or notes are allowed.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
- Do not use your own scratch paper. Extra scratch paper is available from the front of the room.
- **IF YOU HAVE EXTRA TIME, YOU SHOULD CHECK YOUR ANSWERS.**

Question	Points	Score
1	30	
2	30	
3	30	
4	30	
5	30	
6	25	
7	25	
Total:	200	

1. (a) (10 points) Find the general real-valued solution for the following system of equations:

$$\begin{aligned}y_1' &= 3y_1 + y_2 \\y_2' &= y_1 + 3y_2\end{aligned}$$

(b) (5 points) Find the particular solution for the system in part (a) which satisfies the initial conditions  $y_1(0) = 1, y_2(0) = 0$ .

(c) (15 points) Which graph corresponds to this solution? (A, B, C, D, E, F, G, or H)

2. (30 points) Suppose that there are four tanks: tank 1, tank 2, tank 3, and tank 4.

Tank 1 initially contains 1 pound of salt and 1 gallon of water. Tank 2 initially contains 2 pounds of salt and  $\frac{1}{2}$  gallon of water. Tank 3 initially contains 3 pounds of salt and  $\frac{1}{3}$  gallon of water, Tank 4 initially contains 4 pounds of salt and  $\frac{1}{4}$  gallon of water.

Fresh water flows into tank 1 at a rate of 2 gallons per second. Salt solution leaves tank 4 and flows into the drain at 2 gallons per second. Salt Solution flows from tank 1 to tank 2 at a rate of 1 gallon per second, and from tank 1 to tank 3 at a rate of 1 gallon per second. Salt solution flows from tank 2 to tank 4 at 1 gallon per second and from tank 3 to tank 4 at 1 gallon per second.

Find a system of equations and initial conditions describing this problem. **Do not solve the system of equations.**

3. (30 points) Find the general real-valued solution to the three-dimensional system of equations  $y' = Ay$  where

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

4. (30 points) Find the general real-valued solution to the three-dimensional system of equations  $y' = Ay$  where

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

5. (30 points) Find the general real-valued solution to the three-dimensional system of equations  $y' = Ay$  where

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 1 \\ 0 & 0 & 8 \end{pmatrix}$$

6. (25 points) Find the general real-valued solution for

$$t^2 \frac{dy}{dt} + (y^2 - ty) = 0.$$

*Hint:* There is an integrating factor  $\mu$  of the form  $\mu(t, y) = 1/(t^a y^b)$

7. (25 points) Find the general real-valued solution for

$$y'' - 4y' + 4y = (\cos^2(t) - \sin(t))e^{2t+\sin(t)}$$

*Hint:*  $\frac{d}{dt} \cos(t)e^{\sin(t)} = (\cos^2(t) - \sin(t))e^{\sin(t)}$

Extra Scratch Paper:

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