

Name: _____

Instructions:

- There are 4 problems. Make sure you are not missing any pages.
- Unless stated otherwise (or unless it trivializes the problem), you may use without proof anything proven in the sections of the book covered by this test (excluding the exercises).
- Give complete, convincing, and clear answers (or points will be deducted).
- No calculators, books, or notes are allowed.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (10 points) Suppose that $a \leq b$ and $(x_n)_{n=1}^{\infty}$ is a convergent sequence with $x_n \in [a, b]$ for every n (here $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$). Prove that $\lim_{n \rightarrow \infty} x_n \in [a, b]$.

2. (10 points) Suppose that $(x_n)_{n=1}^{\infty}$ is a sequence satisfying $|x_n - x_{n+1}| \leq 2^{-n}$ for every n . Prove that $(x_n)_{n=1}^{\infty}$ is a Cauchy sequence.

3. (10 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined $f(x) = 0$ if x is irrational and $f(x) = x$ if x is rational. Show that f is continuous at 0.

4. (10 points) Let $(x_n)_{n=1}^{\infty}$ be a convergent sequence. Prove (using only the definitions) that $(x_n)_{n=1}^{\infty}$ is a Cauchy sequence.

Extra Scratch Paper: