

Name: _____

Instructions:

- There are 4 problems. Make sure you are not missing any pages.
- Give complete, convincing, and clear answers (or points will be deducted).
- No calculators, books, or notes are allowed.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (10 points) Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be functions satisfying $(g \circ f)(x) = x$ for all $x \in A$. Prove that f is one-to-one. Must f be onto B ? Justify your answer.

2. (10 points) Let J be a nonempty subset of \mathbb{R} that has the following properties:

(i) J is bounded

(ii) $\sup J \in J$

(iii) $\inf J \notin J$

(iv) If $x, y \in J$ with $x < y$ then $t \in J$ for every t satisfying $x < t < y$.

Prove that J is the interval $(\inf J, \sup J]$.

3. (10 points) Let A and B be nonempty subsets of the positive real numbers which are bounded above. Prove that $\sup\{a + b : a \in A, b \in B\} = (\sup A) + (\sup B)$.

4. (10 points) For each positive integer k let

$$\mathbb{N}^k = \{(x_1, \dots, x_k) : x_1, \dots, x_k \in \mathbb{N}\}$$

be the set of ordered k -tuples of positive integers. Prove that for every positive integer k , \mathbb{N}^k is countable. You are allowed to use, without proof, the fact that $\mathbb{N} \times \mathbb{N}$ is countable. You are also allowed to use, without proof, the fact that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are both bijective, then $g \circ f : A \rightarrow C$ is bijective. *Hint:* Try using induction on k .

Extra Scratch Paper: