Math 131A Lecture 2
Homework 4 – Due Apr. 24

Note: Group work is permitted on the homework provided that each person writes their own answer in their own words. However, to obtain any major benefit from the homework, it is recommended that the student spend significant time and effort attempting each problem before collaborating or seeking help from the teacher/teaching assistant/other students.

Problem 1: (The Squeeze theorem) Suppose that $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are convergent sequences such that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$ and such that for every n, $a_n \leq b_n$. Prove that if $(c_n)_{n=1}^{\infty}$ is a sequence with $a_n \leq c_n \leq b_n$ for every n then $(c_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} c_n = \lim_{n\to\infty} a_n$.

Problem 1: (Limit of averages) Let $(a_n)_{n=1}^{\infty}$ be a sequence, and let $(s_n)_{n=1}^{\infty}$ be the sequence defined $s_n = \frac{1}{n}\sum_{j=1}^{n} a_j$. Prove that if $(a_n)_{n=1}^{\infty}$ is convergent then $(s_n)_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} a_n$. Also, give an example where $(s_n)_{n=1}^{\infty}$ is convergent, but $(a_n)_{n=1}^{\infty}$ is not convergent.

From the textbook:
Section 2.7: #7.4 (don't need to prove convergence).
Section 2.9: #9.1b,9.5,9.12,9.15.