

Math 131A Lecture 2 Homework 2b – Due Apr. 10

Note: Group work is permitted on the homework provided that each person writes their own answer in their own words. However, to obtain any major benefit from the homework, it is recommended that the student spend significant time and effort attempting each problem before collaborating or seeking help from the teacher/teaching assistant/other students.

From the book: 1.3.7, 1.3.8.

Problem 5: Suppose $f : A \rightarrow B$ is a function. For any set $C \subset A$ denote $f(C) = \{f(x) : x \in C\}$ and for any set $D \subset B$ denote $f^{-1}(D) = \{x \in A : f(x) \in D\}$ (note: these are standard notations, and f does not have to be invertible for $f^{-1}(D)$ to make sense). Prove that for any $D_1, D_2 \subset B$ we have $f^{-1}(D_1 \cap D_2) = f^{-1}(D_1) \cap f^{-1}(D_2)$. Is it also true that for any $C_1, C_2 \subset A$ we have $f(C_1 \cap C_2) = f(C_1) \cap f(C_2)$?

Problem 6: By constructing an explicit map, show that the integers are countable.

Challenge Problem: (*do not turn in*) Let Ω be the set whose elements are the subsets of \mathbb{N} . Show that Ω is not countable (*hint:* One can show this directly using Cantor's diagonal argument. Alternately, use the fact that the set of real numbers is uncountable.)