Math 131A Lecture 2
Homework 2a – Due Apr. 10

Note: Group work is permitted on the homework provided that each person writes their own answer in their own words. However, to obtain any major benefit from the homework, it is recommended that the student spend significant time and effort attempting each problem before collaborating or seeking help from the teacher/teaching assistant/other students.

Problem 1: Suppose that A and B are sets prove the following two distributive laws for unions and intersections (*hint: often, a convenient way to prove $C = D$ is to prove $C \subset D$ and $D \subset C$*).

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

and

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Problem 2: Suppose that R is a set and that Ω is a collection of sets (i.e. a sets whose elements are sets). Prove De Morgan's laws:

$$R \setminus \left(\bigcup_{A \in \Omega} A \right) = \bigcap_{A \in \Omega} (R \setminus A)$$

and

$$R \setminus \left(\bigcap_{A \in \Omega} A \right) = \bigcup_{A \in \Omega} (R \setminus A)$$

(note: when we write $\bigcup_{A \in \Omega} (R \setminus A)$ we mean $\bigcup_{A \in \Omega'} A$ where $\Omega' = \{R \setminus A : A \in \Omega\}$, and similarly for intersections).

Problem 3: Suppose that $f : A \to B$ is a function and $g : B \to C$ is a function. The composition of g with f, denoted $g \circ f$ is defined $(g \circ f)(x) = g(f(x))$. Prove the following three statements.

$$g \circ f : A \to C \quad \text{(i.e. } g \circ f \text{ is a function from } A \text{ into } C\text{)}$$

$$\text{if } g \circ f \text{ is injective then } f \text{ is injective}$$

$$\text{if } g \circ f \text{ is surjective then } g \text{ is surjective}$$

Problem 4: Suppose that $f : A \to A$ and $g : A \to A$ are functions. Then g is said to be an inverse for f is $g \circ f$ is the identity map (i.e. $(g \circ f)(x) = x$ for every $x \in A$) and $f \circ g$ is the identity map (note that g is an inverse for f if and only if f is an inverse for g). Given $f : A \to A$ prove that

$$f \text{ is bijective} \Leftrightarrow \text{there exists a function } g : A \to A \text{ such that } g \text{ is an inverse for } f$$

hint: use problem 3