In a famous paper, Hirzebruch and Zagier considered families of homology classes \( \{Z_m\}_{m \in \mathbb{Z}_{\geq 0}} \) on certain Hilbert modular surfaces and showed that the generating series \( \sum_{n=0}^{\infty} Z_m \cdot Z_n q^n \) are elliptic modular forms with nebentypus. This work can be seen as giving a geometric interpretation of the Doi-Naganuma lifting.

We prove the modularity of analogous generating series in the context of intersection homology classes on the product of two Hilbert modular varieties of arbitrary dimension or a single Hilbert modular variety of arbitrary dimension. The results we will present in the latter situation are joint with M. Goresky. The aim of this work-in-progress is to give a geometric/topological interpretation of abelian base change.