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Title: INTERSECTIVE SETS

Abstract: Let \( h(x) \in \mathbb{Z}[x] \) be a nonconstant polynomial which has a root modulo \( m \) for every positive integer \( m \). Kamae and Mendes France have shown that given any set \( A \) of positive integers with positive upper density there exists two distinct elements \( a, a' \in A \) such that

\[
a - a' = h(x)
\]

for some integer \( x \geq 1 \). Over the years several quantitative versions of this result have been given for specific polynomials. We give a quantitative result which applies to all such polynomials and from which we can deduce the following result. Given \( A \) and \( h(x) \) as above we define \( R(A_N) \) to be the number of solutions of

\[
a - a' = h(x)
\]

with \( a, a' \in A \cap \{1, \ldots, N\} \) and \( x \geq 1 \). If the degree of \( h(x) \) is \( k \geq 2 \), then

\[
\limsup_{N \to \infty} \frac{R(A_N)}{N^{1+1/k}} > 0.
\]

This generalizes a result due to R.C. Vaughan.