NUMBER THEORY SEMINAR
Tuesday, April 27, 4.30 - 5.30 pm, MS 6221

Speaker: Brian Conrad, U. Michigan

TITLE: Root numbers and ranks over global function fields.

ABSTRACT: A useful (unproved) consequence of the Birch and Swinnerton-Dyer conjecture is that the global root number for an elliptic curve over a global field determines the parity of the rank of the Mordell-Weil group. Standard conjectures in analytic number theory imply that if $E \to \mathbb{P}^1_{\mathbb{Q}}$ is a non-isotrivial minimal elliptic surface admitting points of multiplicative reduction on $\mathbb{P}^1_{\mathbb{Q}}$ then the average of the root numbers of the $\mathbb{Q}$-rational fibers is 0; in particular (if one accepts these standard conjectures), infinitely many fibers have root number 1 and infinitely many fibers have root number -1. There have been counterexamples constructed when the nonisotriviality or multiplicative-reduction conditions are dropped.

One of these "standard conjectures", called Chowla's conjecture, is not true for global function fields, and this suggests that there may exist non-isotrivial elliptic surfaces over $\mathbb{P}^1_F$ for a global function field F such that there is a closed point on $\mathbb{P}^1_F$ where the family has multiplicative reduction but the average of the root numbers of the non-degenerate F-rational fibers is not 0. We do better: in the case $F = k(u)$ for varying finite fields k of char $p > 3$, we construct a 4-parameter algebraic family of such elliptic surfaces whose generic fiber (in the Grothendieck sense) has Mordell-Weil group with odd rank 1 but whose F-rational fibers all have global root number $+1$ (and thus have even positive rank, under the parity conjecture). The determination of the rank of the generic fiber rests on a mixture of cohomological and geometric methods, as well as a computation of a Cassels-Tate pairing.

This talk summarizes joint work with K. Conrad and H. Helfgott.