Midterm 1
UCLA: Math 3B, Winter 2017

Instructor: Noah White
Date: 30 January 2017

- This exam has 5 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: Solutions.

ID number: 

Discussion section (please circle):

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Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

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1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) The function $f(x) = \frac{1}{1+x}$ has
   A. a vertical asymptote at $x = -1$.
   B. a horizontal asymptote at $y = 1$.
   C. no asymptotes.
   D. a slanted asymptote.

(b) (1 point) The function $f(x) = \frac{1}{x} - x$ is
   A. increasing when $x > 0$ and decreasing when $x < 0$.
   B. increasing when $x < 0$ and decreasing when $x > 0$.
   C. always increasing.
   D. always decreasing.

(c) (1 point) The function $f(t) = t - 2 \ln t$ has a
   A. local minimum at $t = 2$.
   B. local maximum at $t = 2$.
   C. local minimum at $t = 1$.
   D. local maximum at $t = 1$. 
(d) (1 point) The function \( f(x) = 2 \ln(e^x + 1) - x \) has
   A. a horizontal asymptote at \( y = 1 \).
   B. a vertical asymptote at \( x = 0 \).
   C. no asymptotes.
   \( \boxed{D} \) a slanted asymptote of \( y = x \).

*Hint: For this question you may use the fact that \( \lim_{x \to \infty} f(x) = \infty \) and \( f'(x) = \frac{e^x - 1}{e^x + 1} \)*

(e) (1 point) The function \( h(t) = 1 - \sin x + x \) is an antiderivative of
   A. \( x - \cos x + 0.5x^2 \)
   \( \boxed{B} \) \( 1 - \cos x \)
   C. \( 1 - \sin x \)
   D. \( x + \cos x - x^2 \)

(f) (1 point) The definite integral \( \int_{-1}^{1} 2x e^{x^2} \, dx \)
   A. 2
   B. \( \frac{1}{2} (e - 1) \)
   C. \( e - 1 \)
   \( \boxed{D} \) 0
2. Let $f(x) = \frac{1}{x^2 - x - 2}$. Note that $f'(x) = \frac{1 - 2x}{(x-2)^2(x+1)^2}$ and $f''(x) = \frac{6(x^2 - x + 1)}{(x-2)^3(x+1)^3}$.

(a) (2 points) Find the $x$ and $y$ intercepts of $f(x)$.

$f(0) = -\frac{1}{2}$  $y$-int:  $y = -\frac{1}{2}$

$f(x) \neq 0$ so no $x-$int.

(b) (1 point) Does $f(x)$ have any horizontal asymptotes? If so what are they?

$$\lim_{x \to \pm \infty} f(x) = 0$$ so hor. asymptote at $y=0$ in both directions.

(c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

$f(x)$ undefined at $x = 2, -1$, yes, vert. asymptotes @ $x = 2, -1$.

$$\left(\lim_{x \to 2^+} f(x) = \infty, \lim_{x \to 2^-} f(x) = -\infty, \lim_{x \to -1^+} f(x) = -\infty, \lim_{x \to -1^-} f(x) = +\infty\right)$$

(d) (2 points) For what $x$ is the first derivative $f''(x)$ positive?

$f''(x) = 0$ when $x = \frac{1}{2}$, undefined $x = 2, -1$
(e) (2 points) For what $x$ is the second derivative $f''(x)$ positive?

\[ f''(x) \neq 0 \quad \text{since} \quad x^2 - x + 1 \neq 0 \]
\[ \text{(discriminant} = -3) \quad f'' \text{ undefined} \quad x = 2, -1. \]

(f) (4 points) On the graph provided, sketch $f(x)$
3. A company would like to advertise its product in two media markets (market A and market B). It is known experimentally that a spend of $\$x$ thousand dollars in market A will result in

$$R_A(x) = 18 \ln(1 + x)$$

thousand extra customers and a spend of $\$y$ thousand in market B will result in

$$R_B(x) = 8 \ln(1 + x)$$

thousand new customers.

(a) (4 points) If the company has a marketing budget of $\$10$ thousand and they wish to spend it all, how many new customers will they attract in total? (Let $\$x$ thousand be the amount spent in market A.)

\[
x = \text{s pend in A} \\
y = \text{s pend in B} \\
\text{so } x + y = 10
\]

Total new customers:

\[
T = R_A(x) + R_B(y)
\]

But $y = 10 - x$ so

\[
T(x) = R_A(x) + R_B(10 - x)
\]

\[= 18 \ln(1 + x) + 8 \ln(11 - x)\]
(b) (5 points) How much should the company spend in market $A$ in order to maximise the number of new customers they attract?

$$T'(x) = \frac{18}{1+x} - \frac{8}{11-x}$$

Find critical points $T'(x) = 0$

$$\frac{18}{1+x} = \frac{8}{11-x}$$

$$18(11-x) = 8(1+x)$$

$$9(11-x) = 4(1+x)$$

$$99 - 9x = 4 + 4x$$

$$95 = 13x$$

$$x = \frac{95}{13}.$$

Domain is $[0, 10]$. Use closed int. test

$$T\left(\frac{95}{13}\right) \approx 48.6$$

$$T(0) \approx 19.2$$

$$T(10) \approx 43.2$$

So $x = \frac{95}{13}$ is a global max.
5. A trough is being built for feeding animals. The cross section of the is pictured below.

Each side, and the base of the trough are 1 foot wide. The sides are bent up from the horizontal, at an angle $\theta$.

(a) (3 points) What is the area of the cross section?

\[
\text{Break into two triangles + rectangle}
\]

\[
\text{height } = \sin \theta \\
\text{base of triangle } = \cos \theta
\]

\[
A = \text{Area} = 2 \left( \frac{1}{2} \sin \theta \cos \theta \right) + 1 \cdot \sin \theta = \sin \theta + \sin \theta \cos \theta.
\]

(b) (5 points) What angle should the sides be bent to, in order that the trough can hold as much food as possible? You may use the fact that $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$.

Maximise $A$ on the domain $[0, \pi/2]$.

\[
A'(\theta) = \cos \theta + \cos \theta - \sin^2 \theta = 2 \cos^2 \theta + \cos \theta - 1
\]

Find critical points: factorise quadratic

\[
(2 \cos \theta - 1)(\cos \theta + 1) = 0
\]

so \( \cos \theta = \frac{1}{2} \) or \(-1\)

so \( \theta = \frac{\pi}{3} \) or \( \pi \) --- not in domain

End points $\theta = 0, \frac{\pi}{2}$, use closed int. method

\[
A(0) = 0 \quad A\left(\frac{\pi}{2}\right) = 1 \quad A\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{4} > 1
\]

so \( \theta = \frac{\pi}{3} \) is the global max.
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