Last time, we spoke about

- Graphing using calculus
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- Examples
A function is three pieces of information
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- A domain, $D \subset \mathbb{R}$
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- A range, $R \subset \mathbb{R}$, and
A function is three pieces of information

- A domain, \( \mathbb{D} \subset \mathbb{R} \)
- A range, \( \mathbb{R} \subset \mathbb{R} \), and
- A rule \( f : \mathbb{D} \rightarrow \mathbb{R} \) that assigns to every element of \( D \) an element of \( R \).
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**Example**
The functions

are all different functions!
Functions

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Example

The functions

- \( f : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto x^2 \)

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**Example**

The functions

- $f : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto x^2$
- $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}; x \mapsto x^2$

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Global Maximums and minimums

Definition (Global maximum)
A function $f : D \rightarrow R$ has a global maximum at $a$ if

$$f(x) \leq f(a) \quad \text{for all } x \in D$$
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\]
Example of a global minimum

\[ f : \mathbb{R} \rightarrow \mathbb{R}; \ x \mapsto x^2 \] has a min at \( x = 0 \)
Example of a global maximum

\[ f : (-\infty, 0] \rightarrow \mathbb{R}; f(x) = x^3 \text{ has a max at } x = 0 \]
Local Maximums and minimums

**Definition (local maximum)**
A function $f : D \rightarrow R$ has a local maximum at $a$ if

$$f(x) \leq f(a) \quad \text{for all } x \text{ near } a$$

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Example of a local minimum

\[ f : \mathbb{R} \longrightarrow \mathbb{R}; \, x \mapsto x^2 \] has a min at \( x = 0 \)
Example of a local maximum

\[ f : \mathbb{R} \rightarrow \mathbb{R}; \quad f(x) = x^3 - 4x^2 - 3x + 13 \text{ has a local max at } x = -\frac{1}{3} \]
Example of a local maximum

\[ f : \mathbb{R} \rightarrow \mathbb{R}; f(x) = x^3 - 4x^2 - 3x + 13 \text{ has a local max at } x = -\frac{1}{3} \]
Definition (Critical point)

A function $f(x)$ has a critical point at $x = a$ if $f'(a) = 0$ or if $f'(a)$ is undefined.
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Examples
- $f(x) = x^2$ has a critical point at $x = 0$ (since $f'(x) = 2x$)
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- \( f(x) = x^2 \) has a critical point at \( x = 0 \) (since \( f'(x) = 2x \))
- \( f(x) = \sin x \) has a critical point at \( x = \frac{\pi}{2} \) (since \( f'(x) = \cos x \))
Definition (Critical point)
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Examples

- $f(x) = x^2$ has a critical point at $x = 0$ (since $f'(x) = 2x$)
- $f(x) = \sin x$ has a critical point at $x = \frac{\pi}{2}$ (since $f'(x) = \cos x$)
- $f(x) = e^x$ doesn’t have any critical points since $f'(x) = e^x$ can never be zero
How to find minimums and maximums

Local maximums and minimums (extrema) occur at
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- critical points
How to find minimums and maximums

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- end points of the domain (are also critical points!)
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**Note:** All extrema are critical points, but not all critical points are extrema!
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Example
\[ f : (-\infty, 1] \rightarrow \mathbb{R}; \quad f(x) = x^3 \] has critical points at
\[ x = 0 \text{ and } 1 \]
Example

\[ f'(x) = 3x^2 \] so \[ f'(0) = 0 \] and \[ f'(1) \] is undefined.
First derivative test

Suppose \( x = a \) is a critical point for the function \( f(x) \).

First derivative test (minimums)
First derivative test

Suppose $x = a$ is a critical point for the function $f(x)$.

First derivative test (minimums)

- If $f'(x) < 0$ for $x$ less than and close to $a$, and
Suppose $x = a$ is a critical point for the function $f(x)$.

**First derivative test (minimums)**

- If $f'(x) < 0$ for $x$ less than and close to $a$, and
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![Graph showing the first derivative test](image)
First derivative test

Suppose \( x = a \) is a critical point for the function \( f(x) \).

First derivative test (maxima)

- If \( f'(x) > 0 \) for \( x \) less than and close to \( a \), and
- \( f'(x) < 0 \) for \( x \) greater than and close to \( a \), then
  - \( f(x) \) has a maximum at \( a \).
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\[ (-10.36, -6.33) \]
\[ (-2.93, -13.64) \]
Second derivative test

Suppose \( x = a \) is a critical point of the function \( f(x) \)

Second derivative test

If

\[
\begin{align*}
\text{If } f''(a) > 0 \text{ then } f \text{ has a minimum at } a \\
\text{If } f''(a) < 0 \text{ then } f \text{ has a maximum at } a \\
\text{Note: If } f''(a) = 0 \text{ then we cannot conclude anything! E.g } x^3 \text{ or } x^4.
\end{align*}
\]
Suppose $x = a$ is a critical point of the function $f(x)$.

**Second derivative test**

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- $f''(a) > 0$ then $f$ has a minimum at $a$.
- $f''(a) < 0$ then $f$ has a maximum at $a$.

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