Autonomous equations

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An ODE of the form
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**Important property**
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We want points \((t, y)\) such that \( f(y) = 0 \).

- Suppose \( f(a) = 0 \).
- Then \((t, a)\) is on the nullcline, for any \( t \).
- So the line \( y = a \) is part of the nullcline, whenever \( f(a) = 0 \).
Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like
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Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to the $y$ axis.
2. Draw dots where equilibrium solutions live.
3. Draw up arrows on intervals between dots where the derivative is positive.
4. Draw down arrows on intervals between dots where the derivative is negative.

Definition

• An equilibrium is stable if the two arrows are pointing towards it.
• It is unstable if the two arrows are pointing away from it.
• It is semistable if the arrows point in the same direction.
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stable
unstable
semistable
semistable
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Example

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- 25 stable
- 10 unstable
- 0 stable
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Classifying equilibria using derivatives

Classification of equilibria

If \( a \) is an equilibrium of

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(i.e. \( f(a) = 0 \)) then \( a \) is

• stable if \( f'(a) < 0 \)
• unstable if \( f'(a) > 0 \)
• indeterminate if \( f'(a) = 0 \)
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