MATH 215A Fall 2017

Homework 6

Due Dec 4

Please turn in the solutions ONLY to the problems marked mandatory (3×10 pts).

Try and try again. And then if needed, look up

Here's Hensel's Lemma for $\mathbb{Z}_p$: Let $f(x) \in \mathbb{Z}_p[x]$ and $a \in \mathbb{Z}_p$ such that $|f(a)| < |f'(a)|^2$ where $|x| = \frac{1}{p}^{v_p(x)}$ is the absolute value induced by the p-adic valuation v_p. Then there is a unique $b \in \mathbb{Z}_p$ such that $f(b) = 0$ and $|b - a| < |f'(a)|$.

1. Relate this to the Hensel's Lemma stated in class.
2. Prove it by imitating Newton's method as follows: Define sequence $a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$ and set $t = \frac{|f(a)|}{|f'(a)|^2}$. Show that each $a_n \in \mathbb{Z}_p$ (i.e. $|a_n| \leq 1$), $|f'(a_n)| = |f'(a_1)|$ and $|f(a_n)| \leq |f'(a_1)|^2 t^{2^{n-1}}$. Show $\{a_n\}$ is Cauchy. Let b be its limit...

Mandatory problems

1. Find all roots of unity in $\mathbb{Q}_p$.
2. Let $f(x) = x^4 - 7x^3 + 2x^2 + 2x + 1$. How many roots does it have in $\mathbb{Z}_3$. Find their 3-adic expansions.
3. Let R be a ring complete with respect to an ideal I. Show that I is contained in the Jacobson radical of R.