MATH 215A Fall 2017

Homework 4

Due Nov 15

Please turn in the solutions ONLY to the problems marked mandatory (3×10 pts).

Practice problems

1. Let X be the interval $[-1, 1] \subseteq \mathbb{R}$ with the usual subspace (Euclidean) topology and $C(X)$, the set of real valued continuous functions on X. $C(X)$ is a ring by $f \times g(x) = f(x)g(x)$ and $(f+g)(x) = f(x) + g(x)$. Show that $C(X)$ is not noetherian. [Hint : Look at strictly decreasing chain of closed sets of X and look at functions which vanish on them..]

2. Let R be a commutative ring. Show that if $R[x]$ is noetherian, so is R

3. Show that if R is a local ring and M is an R-module with composition series $M = M_0 \supset M_1 \ldots M_n = (0)$, then $M_i/M_{i+1} \simeq M_j/M_{j+1}$ for any i, j.

Try and try again. And then if needed, look up

1. A topological space X is called Noetherian if the open subsets of X satisfy the ascending chain condition (or equivalently, if the closed sets of X satsify dcc). Show that a noetherian space is a finite union of irreducible closed subspaces. If A is a noetherian ring, show $Spec(A)$ is a noetherian topological space. What about the converse ?

2. Let R be a noetherian ring and $R[[x]]$, the ring of formal power series in x with coefficients in R. Show that $R[[x]]$ is Noetherian. [Hint : Adapt the proof of Hilbert basis theorem suitably]

3. Problem 5 in Chapter 7, Atiyah MacDonald.

Mandatory problems

1. Find a noether normalization of $\mathbb{C}[x, y, z]/(xy + z^2, x^2y - xy^3 + z^4 - 1)$

2. Let M be a nonzero module over a noetherian ring R. Show that there is an ideal P in R which is maximal among the annihilators of nonzero elements of M. Show that such an ideal P must be prime.

3. Let R be a local Noetherian ring with unique maximal ideal m. Given m is principal (i.e, generated as an ideal by a single element), then show that every ideal of R is principal [Hint : If not, pick an ideal maximal among non-principal ideals... and roll with it!]