MATH 215A Fall 2017

Homework 3

Due Nov 1

Please turn in the solutions ONLY to the problems marked mandatory (3×10 pts).

Practice problems

1. Let $A = k[x,y]/(y^2 - x^2 - x^3)$ where k is a field. Find the integral closure of A in its fraction field K.

2. Let n be a square free positive integer. Find the integral closure of $\mathbb{Z}$ in $\mathbb{Q}(\sqrt{n})$.

3. Let A be a subring of B such that B is a finitely generated A-module. Show that if A is a field, B has only finitely many prime ideals. Hence show that in the general case (when A is not neccesarily a field), given a prime ideal P of A, there exists only finitely many prime ideals Q of B with $Q \cap A = P$.

Try and try again. And then if needed, look up

1. A ring homomorphism $f : A \to B$ is said to have going down property if the conclusion for the going down theorem holds for B and subring $f(A)$. Let $f : A \to B$ be a flat morphism of rings (i.e. B is flat as an A-module via f). Show that f has going down property

2. For a geometric interpretation of going up and going down property of ring morphisms, see Problem 10 in Chapter 5, Atiyah MacDonald

3. Problem 15 (about fibers of an integral extension) in Chapter 5, Atiyah MacDonald.

Mandatory problems

1. Let $A = k[x, y]/(y^5 - x^{19})$ where k is a field. Show that A is a domain. Find the integral closure of A in its fraction field K.

2. *Gauss' Lemma* : Let R be an integrally closed domain with quotient field K. Let $f(x) \in R[x]$ be monic and let $f(x) = g(x)h(x)$ for monic polynomials $g(x), h(x) \in K[x]$. Show that $g(x), h(x) \in R[x]$. What happens if R is not integrally closed ?

3. Let $(A, m_A), (B, m_B)$ be two local rings. B is said to dominate A if A is a subring of B and $m_A \subseteq m_B$. Let K be a field and Σ, the poset of all local subrings of K ordered by the relation of domination. Show that Σ has maximal elements and that $A \in \Sigma$ is maximal iff A is a valuation ring of K.