MATH 132 QUIZ 1 (PRACTICE)

(1) Find the range of the function \( f(z) = e^z \) with domain \( \text{Re}(z) < 0 \).

(2) Suppose that \( |\alpha| < 1 \). Prove that \( \left| \frac{z - \alpha}{1 - \bar{\alpha} z} \right| = 1 \) if and only if \( |z| = 1 \).

(3) (a) Find an entire function \( f \) whose real part is \( u(x, y) = x^2 + xy - y^2 \).

(b) Explain why there is no entire function whose real part is \( u(x, y) = y^2 + 2xy \).