MATH 131A HOMEWORK 7

*General hint:* every question uses the intermediate value theorem.

(1) Suppose that $f$ and $g$ are continuous on $[a, b]$ such that $f(a) \geq g(a)$ and $f(b) \leq g(b)$. Prove that there exists a point $c \in [a, b]$ such that $f(c) = g(c)$.

(2) Suppose that $f$ is a real-valued continuous function on $\mathbb{R}$ and that $f(a)f(b) < 0$ for some $a, b \in \mathbb{R}$. Prove that there exists a point $c$ between $a$ and $b$ such that $f(c) = 0$.

(3) Suppose that $f$ is continuous on $[0, 2]$ and that $f(0) = f(2)$. Prove that there exists a point $c \in [0, 2]$ such that $f(c) = f(c + 1)$.

*Hint:* Consider the function $g(x) = f(x + 1) - f(x)$ on the interval $[0, 1]$.