

Quiz #4

Solutions

1. Find the determinant $\begin{vmatrix} 2 & 3 & 0 & 5 \\ 1 & 2 & 1 & 9 \\ 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 1 \end{vmatrix} = 1 \cdot (-1)^{2+3} \begin{vmatrix} 2 & 3 & 5 \\ 1 & 2 & 3 \\ 2 & 4 & 1 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 1 & 2 \\ 2 & 4 \end{vmatrix}$

$$\begin{aligned} &= -1 \left((2 \cdot 2 \cdot 1 + 3 \cdot 3 \cdot 2 + 5 \cdot 1 \cdot 4) - (2 \cdot 2 \cdot 5 + 4 \cdot 3 \cdot 2 + 1 \cdot 1 \cdot 3) \right) \\ &= -1 \left(\underbrace{(4 + 18 + 20)}_{42} - \underbrace{(20 + 24 + 3)}_{47} \right) \\ &= \boxed{5} \end{aligned}$$

2. If A is an $n \times n$ matrix with $|A| = 3$, find the determinants of the following matrices:

- B is the matrix we get by swapping the first two rows of A . $|B| = -3$
- C is the matrix we get by scaling the first row by 2 and the second row by 3. $|C| = 18$
- D is the matrix we get by adding 3 times the second row to the fourth row. $|D| = 3$

3. Find the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

$$p(\lambda) = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 1 = \lambda^2 - 4\lambda + 3 = (\lambda-3)(\lambda-1)$$

$$\text{So } p(\lambda) = 0 \iff \boxed{\lambda = 1, 3}$$