HW3

pg 105 #2.a

Minimize $z = -5x_1 - 7x_2 - 12x_3 + x_4$

subject to

$2x_1 + 3x_2 + 2x_3 + x_4 \le 38$

$3x_1 + 2x_2 + 4x_3 - x_4 \le 55$

$x_i \ge 0$

First put in standard form

$$2x_1 + 3x_2 + 2x_3 + x_4 + x_5 = 38$$
$$3x_1 + 2x_2 + 4x_3 - x_4 + x_6 = 55$$
$$x_i \ge 0$$

$$x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 38 \\ 55 \end{pmatrix}$$

Will use Tableaus

entering variable

$-z$	-5	-7	-12	1	0	0	0
x_5	2	3	2	1	1	0	38
x_6	3	2	4	-1	0	1	55

exiting variable

$38/2 = 19$

$55/4$ ← min

applying Row operations to make 3rd column $\begin{pmatrix}0\\0\end{pmatrix}$ we get

entering variable

$-z$	4	-1	0	-2	0	3	165
x_5	$1/2$	2	0	$3/2$	1	$-1/2$	$21/2$
x_3	$3/4$	$1/2$	1	$-1/4$	0	$1/4$	$55/4$

exiting variable

$\frac{21/2}{3/2} = 7$ ← min

$\frac{55/4}{-1/4} < 0$ X

applying row operations to make 4th column $\begin{pmatrix}0\\0\\1\end{pmatrix}$ we get

$-z$	$14/3$	$5/3$	0	0	$4/3$	$7/3$	179
x_4	$1/3$	$4/3$	0	1	$2/3$	$-1/3$	7
x_3	$5/6$	$5/6$	1	0	$1/6$	$1/6$	$31/2$

Since top column is all positive

$$\Rightarrow \boxed{x = \begin{pmatrix} 0 \\ 0 \\ 31/2 \\ 7 \\ 0 \\ 0 \end{pmatrix} \text{ is optimal at } z = -179}$$

~~#6 If a simplex method has [illegible] exited & then reentered then the basic feasible solution at those two times must have been the same ⇒ z value [illegible] must be the same, but this contradicts the fact that z must decrease at every step. Thus a variable which just exited cannot reenter.~~

(6) Prove a variable which has just left the basis cannot reenter the next iteration.

Suppose X_B are the Basic variables
X_N are the nonbasic variables

& Z is expressed as a function of X_N
X_B is expressed as a function of X_N.
so let X_j be the exiting variable & X_i the entering variable. So we know

$$c_i = \min (C_N) < 0$$

& if $X_B = b - A X_N$ since X_j is the exiting variable

$$X_j = b_j - A_j X_N \quad \& \quad \frac{b_j}{a_{j,i}} = \min_{k \in B} \left(\frac{b_k}{a_{k,i}} : a_{k,i} > 0 \right)$$

So let $\hat{B}$ be our new list of basic variables with X_i removed & X_j added

& lets express Z in terms of $X_{\hat{B}}$ & in particular look at the coefficient of X_j $\hat{c}_j$. Our goal is to show its positive

$$Z = c_1 X_{n_1} + \cdots c_i X_i + \cdots + c_k X_{n_k}$$

$$X_j = b_j - a_{j,1} X_{n_1} \cdots - a_{j,i} X_i \cdots - a_{j,k} X_{n_k}$$

so $$X_i = \frac{b_j}{a_{j,i}} - \frac{a_{j,1}}{a_{j,i}} X_{n_1} \cdots - \frac{X_j}{a_{j,i}} \cdots - \frac{a_{j,k}}{a_{j,i}} X_{n_k}$$

plugging into Z we get

$$z = c_1 x_{n_1} + \dots + c_i \left(\frac{b_j}{a_{j,i}} - \frac{a_{j,1}}{a_{j,i}} x_{n_1} \dots - \frac{x_j}{a_{j,i}} - \dots \frac{a_{j,k}}{a_{j,i}} x_{n_k} \right) + \dots + c_k x_{n_k}$$

$$= \hat{c}_1 x_{n_1} + \dots \hat{c}_j x_j + \dots \hat{c}_k x_{n_k}$$

where $\hat{c}_j = -\dfrac{c_i}{a_{j,i}}$

Now since $c_i < 0$ & $a_{j,i} > 0$

$\Rightarrow \hat{c}_j > 0 \Rightarrow x_j$ can not be chosen

as the new entering variable. ✓