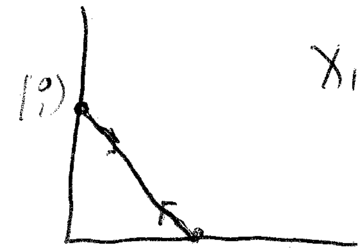
HW1

Pg. 52 (2) ~~(4)~~



$$x_1 + x_2 = 1$$
$$x_1, x_2 \geq 0$$

(a) from $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ feasible directions are $\left\{ \begin{pmatrix} \alpha \\ -\alpha \end{pmatrix} : \alpha > 0 \right\}$

(b) from $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ feasible directions are $\left\{ \begin{pmatrix} -\alpha \\ \alpha \end{pmatrix} : \alpha > 0 \right.$

(3)

$$Ax \geq b$$

$$A = \begin{pmatrix} 9 & 4 & 1 & 9 & -7 \\ 6 & -7 & 8 & -4 & -6 \\ 1 & 6 & 3 & -7 & 6 \end{pmatrix} \qquad b = \begin{pmatrix} -15 \\ -30 \\ -20 \end{pmatrix}$$

(a) $x = \begin{pmatrix} 8 \\ 4 \\ -3 \\ 4 \\ 1 \end{pmatrix} \qquad p = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

Note $Ax = \begin{pmatrix} 114 \\ -26 \\ 1 \end{pmatrix} \geq b$ so x satisfies constraint & is inactive everywhere

$Ap = \begin{pmatrix} 16 \\ -3 \\ 9 \end{pmatrix} \leftarrow$ Only negative entry

so $\bar{\alpha} = \min \left\{ \frac{a_i^T x - b_i}{-a_i^T p} : a_i^T p < 0 \right\} = \min \left(\frac{-26 - (-30)}{-(-3)} \right) = \boxed{\frac{4}{3}}$

(b) $x = \begin{pmatrix} 7 \\ -4 \\ -3 \\ -3 \\ 3 \end{pmatrix}$ $p = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \\ -2 \end{pmatrix}$

$Ax = \begin{pmatrix} -4 \\ 40 \\ 13 \end{pmatrix} > b$ Satisfies Constraints, No active Constraints

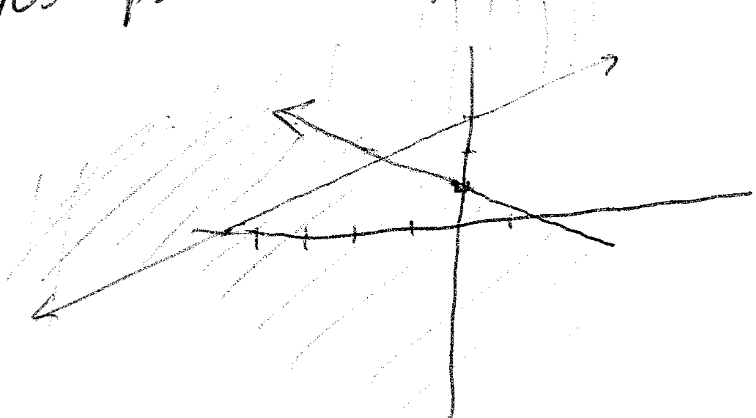
$Ap = \begin{pmatrix} 58 \\ 12 \\ -4 \end{pmatrix} \leftarrow$ only negative

$\bar{\alpha} = \min \left\{ \frac{a_i^T \cancel{x} - b_i}{-a_i^T p} : a_i^T p < 0 \right\} = \min \left\{ \frac{13 - -(20)}{-(-4)} \right\} = \boxed{\frac{33}{4}}$

page 69 1a) Minimize $z = 3x_1 + x_2$

Subject to
$x_1 - x_2 \le 1$
$3x_1 + 2x_2 \le 12$
$2x_1 + 3x_2 \le 3$
$-2x_1 + 3x_2 \ge 9$
$x_1, x_2 \ge 0$

No points satisfy $2x_1 + 3x_2 \le 3$ & $-2x_1 + 3x_2 \ge 9$ & $x_1, x_2 \ge 0$



So there are no feasible points & thus No solution to the problem

page 75 2) Convert

Minimize $z = x_1 - 5x_2 - 7x_3$
subject to
$5x_1 - 2x_2 + 6x_3 \ge 5$
$3x_1 + 4x_2 - 9x_3 = 3$
$7x_1 + 3x_2 + 5x_3 \le 9$
$x_1 \ge -2$, x_2, x_3 free

into standard form

let $x_1' = x_1 + 2$ $\quad x_1 \ge 0$
$x_2' - x_2'' = x_2$ $\quad x_2', x_2'' \ge 0$
$x_3' - x_3'' = x_3$ $\quad x_3', x_3'' \ge 0$

& Add in slack variable x_5 & excess variable x_4 to get

$$5(X_1'-2)-2(X_2'-X_2'')+6(X_3'-X_3'')-X_4=5$$
$$3(X_1'-2)+4(X_2'-X_2'')-9(X_3'-X_3'')=3$$
$$7(X_1'-2)+3(X_2'-X_2'')+5(X_3'-X_3'')+X_5=9$$

giving

$$\underbrace{\begin{pmatrix} 5 & -2 & 2 & 6 & -6 & -1 & 0 \\ 3 & 4 & -4 & -9 & 9 & 0 & 0 \\ 7 & 3 & -3 & 5 & -5 & 0 & 1 \end{pmatrix}}_{A} \begin{pmatrix} X_1' \\ X_2' \\ X_2'' \\ X_3' \\ X_3'' \\ X_4 \\ X_5 \end{pmatrix} = \underbrace{\begin{pmatrix} 15 \\ 9 \\ 23 \end{pmatrix}}_{b}$$

Note $b \geq 0$ $\quad X_1', X_2', X_2'', X_3', X_3'', X_4, X_5 \geq 0$

$$Z = (X_1'-2) - 5(X_2'-X_2'') - 7(X_3'-X_3'')$$
$$= X_1' - 5X_2' + 5X_2'' - 7X_3' + 7X_3'' - 2$$

There should be no constants & minimizing Z is the same as minimizing Z + 2 so

we have $\boxed{\text{Minimize } \hat{Z} = X_1' - 5X_2' + 5X_2'' - 7X_3' + 7X_3''}$