1. Let \( H \) be a \( p \)-subgroup of a finite group \( G \). Show that if \( H \) is not a Sylow \( p \)-subgroup, then \( N_G(H) \neq H \).

2. Let \( G \) be a \( p \)-group and let \( k \) be a divisor of \( |G| \). Prove that \( G \) contains a normal subgroup of order \( k \).

3. Prove that if a group \( G \) contains a subgroup \( H \) of finite index, then \( G \) contains a normal subgroup \( N \) of finite index such that \( N \subseteq H \). (Hint: Consider the action of \( G \) on \( G/H \) by left translations.)

4. Let \( G \) be a \( p \)-group and \( H \) a normal subgroup in \( G \) of order \( p \). Show that \( H \subseteq Z(G) \). (Hint: consider the action of \( G \) on \( H \) by conjugation.)

5. (a) A subgroup \( H \) of \( G \) is called characteristic, if \( f(H) = H \) for every automorphism \( f \) of \( G \). Show that a characteristic subgroup \( H \) is normal in \( G \).

   (b) Prove that if \( K \) is a characteristic subgroup of \( H \) and \( H \) is a characteristic subgroup of \( G \), then \( K \) is characteristic in \( G \).

6. For a group \( G \) set \( G^{(0)} = G \) and \( G^{(i+1)} = [G^{(i)}, G^{(i)}] \) for \( i \geq 0 \). Show that \( G^{(i)} \) is a characteristic subgroup of \( G \).

7. (a) For any two subgroups \( K \) and \( H \) of a group \( G \) denote by \( [K,H] \) the subgroup in \( G \) generated by the commutators \( [k,h] = khk^{-1}h^{-1} \) for all \( k \in K \) and \( h \in H \). Show that if \( K \) and \( H \) are normal in \( G \), then so is \( [K,H] \).

   (b) Prove that \( [G,H] \) is normal in \( G \) for every subgroup \( H \subseteq G \).

8. Let \( G \) be a group and \( Z(G) \) the center of \( G \). Show that if \( G/Z(G) \) is nilpotent, then so is \( G \).

9. Assume that a subset \( S \subseteq G \) of a group \( G \) satisfies \( gSg^{-1} \subseteq S \) for all \( g \in G \). Prove that the subgroup generated by \( S \) is normal in \( G \).

10. Let \( N \) be an abelian normal subgroup in a finite group \( G \). Assume that the orders \( |G/N| \) and \( |\text{Aut}(N)| \) are relatively prime. Prove that \( N \) is contained in the center of \( G \).