HOMEWORK 4

1. Find all conjugacy classes in S_n, $n \leq 4$.

2. Find all subgroups in A_4. Show that A_4 has no subgroup of order 6.

3. (a) Prove that S_n is generated by $(1,2)$, $(1,3), \ldots (1,n)$.
(b) Prove that S_n is generated by two cycles $(1,2)$ and $(1,2,\ldots,n)$.

4. Show that A_n $(n \geq 4)$ and S_n $(n \geq 3)$ have trivial centers.

5. (a) Show that the centralizer of A_n in S_n (the subgroup in S_n consisting of all elements, which commute with all elements in A_n) is trivial, if $n \geq 4$.
(b) Let $g \in S_n$ be an odd transformation. Show that the map $f : A_n \to A_n$, given by $f(x) = gxg^{-1}$, is an automorphism. Prove that f is not inner automorphism if $n \geq 3$.

6. Show that $Aut(S_3)$ consists of only inner automorphisms and is isomorphic to S_3.

7. Describe all Sylow subgroups in S_5.

8. Show that every subgroup in S_n of index n is isomorphic to S_{n-1}. (Hint: For a subgroup $H \subset G = S_n$ consider the homomorphism $S_n \to S(G/H)$ induced by the action of G on G/H by left translations.)

9. (a) Show that for $n \geq 5$, any nontrivial subgroup in A_n has index $\geq n$. (Hint: See hint to problem 8.)
(b) Prove that there are no injective homomorphisms $S_n \to A_{n+1}$ for $n \geq 2$.

10. (a) Show that there is an injective homomorphism $S_n \to A_{n+2}$.
(b) Prove that any finite group is isomorphic to a subgroup of a finite simple group.