Math 110C

Take home Final Exam,

due Friday June 9, 2017

Name:

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1. Determine a splitting field of $X^6 - 4$ and its degree over $\mathbb{Q}$.

2. Let $f \in \mathbb{Q}[X]$ be an irreducible polynomial of degree 3. Assume that f has a unique real root α. Show that the extension $\mathbb{Q}(\alpha)/\mathbb{Q}$ is not normal.

3. Let p be a prime integer, $\xi \in \mathbb{C}$ primitive p-th root of unity. Find the degree of the extension $\mathbb{Q}(\xi + \xi^{-1})/\mathbb{Q}$.

4. Let I be the ideal in $\mathbb{Q}[X, Y]$ generated by $X^2 - Y$ and $Y^2 + 4$. Is the factor ring $\mathbb{Q}[X, Y]/I$ a field?

5. Determine the degree of a splitting field of the polynomial $X^7 - 1$ over the finite field $\mathbb{F}_5$.

6. Determine the Galois group of the polynomial $X^4 + 3X^2 + 1$ over $\mathbb{Q}$.

7. Determine the Galois group of the normal closure of the extension $\mathbb{Q}(\sqrt{-3} + \sqrt[3]{2})/\mathbb{Q}$.

8. Let $\alpha = 1 + \sqrt[3]{2} + \sqrt[3]{4}$. Determine the Galois group of the normal closure of the extension $\mathbb{Q}(\alpha)/\mathbb{Q}$.

9. Let p be a prime integer. Suppose that the degree of every finite extension of a field F is divisible by p. Prove that the degree of every finite extension of F is a power of p.

10. Let E_1 and E_2 be two normal extensions of F. Suppose that E_1 and E_2 are subfields of some field. Prove that the intersection $E_1 \cap E_2$ is also a normal extension of F.